Testing Gravitational Redshift through Simulation based on the China Space Station Laser Timing Experiment

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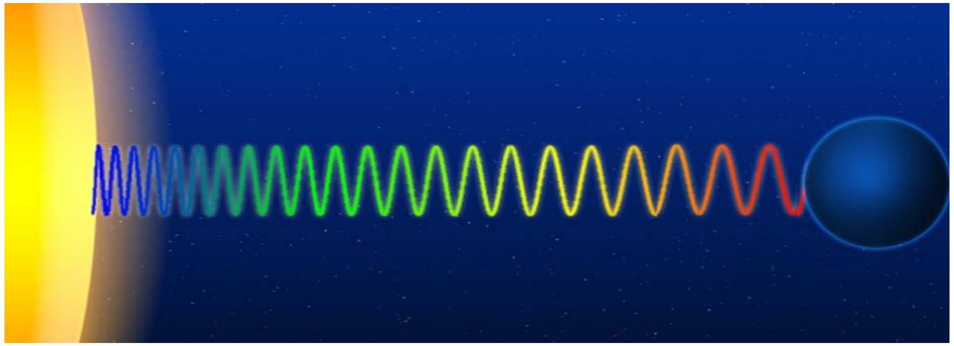
Contents

1. Introduction

▪ Gravitational redshift (GRS)

GRS is relative frequency/time shift between two clocks due to the gravitational field of a body (Will, 2014).

$$\underbrace{\boxed{\frac{\Delta f}{f}}}_{\text{Measure}} = (1+\alpha)\underbrace{\boxed{\frac{\Delta U}{c^2}}}_{\text{Predicted}} \qquad (1)$$



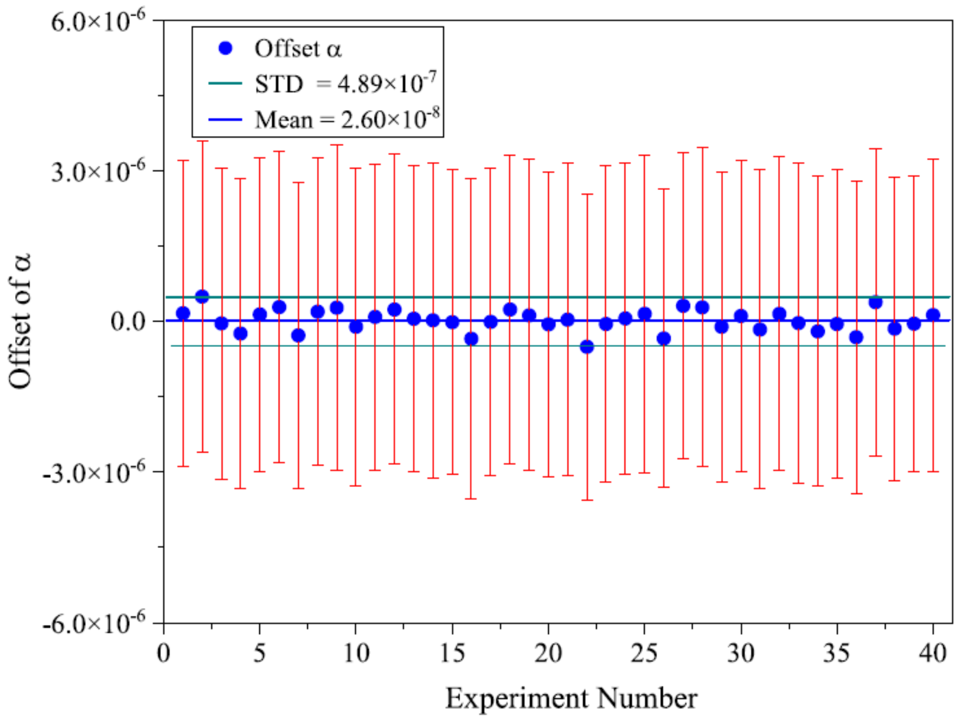
Gravitational redshift
https://en.wikipedia.org/wiki/Gravitational_redshift

If Einstein's is correct, coefficient **$\alpha = 0$**

Several experiments and observations have been conducted to confirm this effect (Pound & Rebka 1960; Hafele & Keating 1972; Vessot et al. 1980; Delva et al., 2018; Cacciapuoti et al, 2020; Shen et al., 2021; 2023)

1. Introduction.

- **Recent GRS Experiment based on Microwave Link aboard CSS**



PHYSICAL REVIEW D **108,** 064031 (2023)

Testing gravitational redshift based on microwave frequency links onboard the China Space Station

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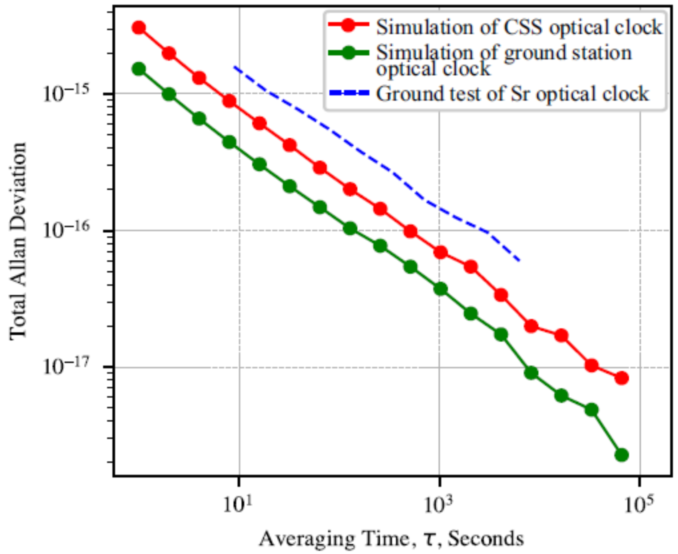
(Received 7 December 2021; accepted 27 June 2023; published 15 September 2023)

In November 2022, the China Space Station (CSS) was equipped with a cold atomic microwave clock and a Sr optical lattice clock with stabilities of $5 \times 10^{-14}/\sqrt{\tau}$ and $2 \times 10^{-15}/\sqrt{\tau}$ (where τ is the integration time in seconds), respectively, which provides an excellent opportunity to test gravitational redshift (GRS) with higher accuracy than previous results. Based on high-precision frequency links between the China Space Station and a ground station, we formulated a model and provided simulation experiments to test GRS. Simulation results suggest that this method could test the GRS at the accuracy level of 5×10^{-7}, more than 2 orders in magnitude higher than the result of the experiment of a hydrogen clock on board a flying rocket more than 40 years ago.

DOI: 10.1103/PhysRevD.108.064031



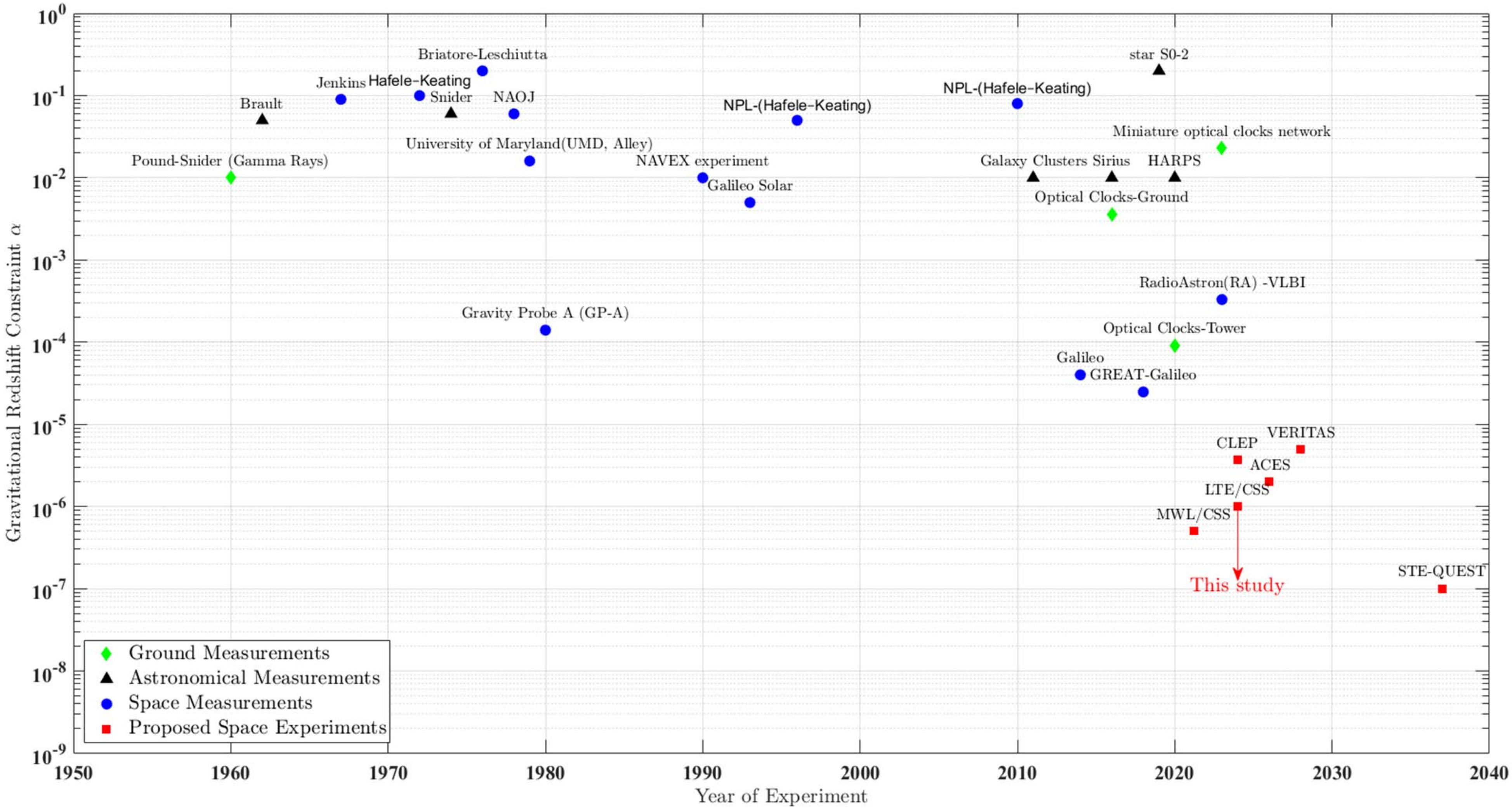
Predicted accuracy of testing GRS

$\boldsymbol{\alpha \approx 5 \times 10^{-7}}$ **level**

(Shen et al., 2023, PRD)

1. Introduction

▪ GRS Experiments

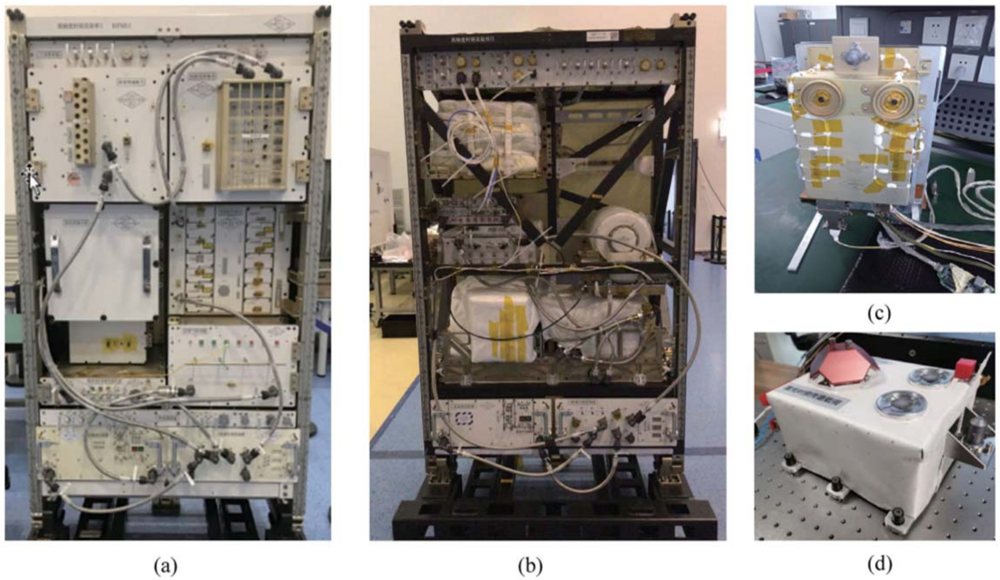


▪ Dramatic improvements in atomic clock technology have enabled much **more sensitive** tests of GRS

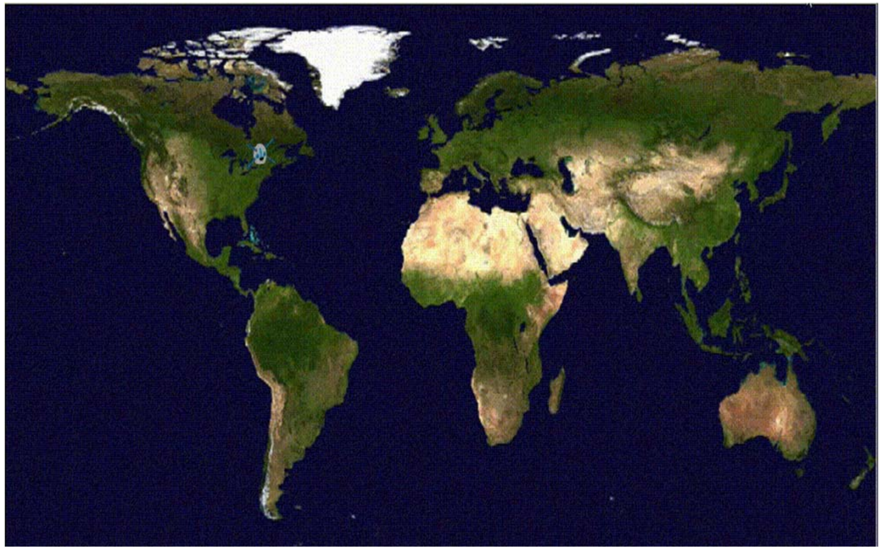
1. Introduction

▪ China Space Station (CSS)

- In October 2022, China successfully installed the first high-precision optical clock in space ($\sim 3 \times 10^{-17}$) aboard its Tiangong space station (Mengtian module).



(a) optical lattice clock (b) cold atom microwave clock, an active hydrogen clock (c) microwave signals emitting and receiving equipment. (d) The payload of laser time-frequency transfer. (Shen et al., 2023, PRD)



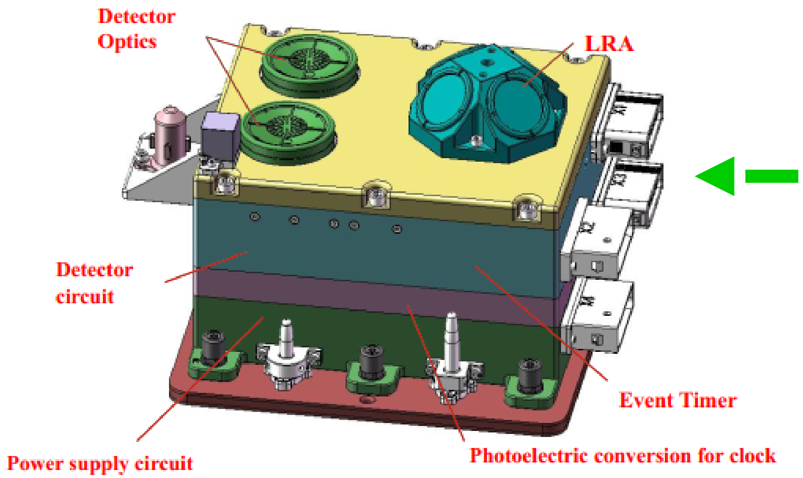
Orbital inclination: 42° ~ 43 ° & altitude: 400 ~ 450 km

Applications:

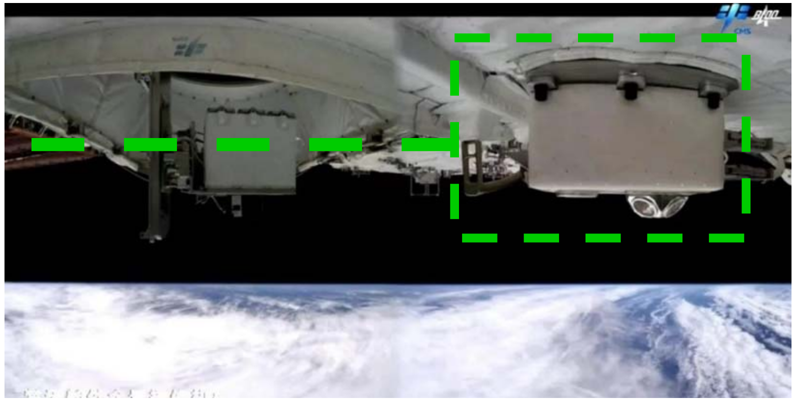
- Tests of Fundamental Physics
- High-precision gravitational potential of Earth from space
- Future deep space exploration
- …etc.

1. Introduction

▪ Equipment of Laser Timing Experiment (LTE) aboard CSS



Design of space LTE payload on CSS (Wu et al., 2022)



- The mass of LTE payload : 6 kg
- Wavelength : 532 nm
- Repetition rate : 1 kHz, 2 kHz, 10 kHz
- Event timer precision: ~8ps
- Time deviation : 0.08 ps/300 s, 0.8 ps/86400 s
- Detector precision: ~25 ps
- Clock reference: 200 MHz (optical comb)

(Geng et al., 2023)

Table (1): High-precision time and frequency standards

Atomic Clocks	Stability
Optical Cold	3×10^{-17}@ 1 day
Hydrogen	2×10^{-15}@ 1 day
Microwave cold	2×10^{-16}@ 1 day

- Additionally, a microwave frequency link is onboard CSS for further details Shen et al. (2023, PRD).

2. Methodology

▪ Two-way laser time transfer (TWLTT) Link

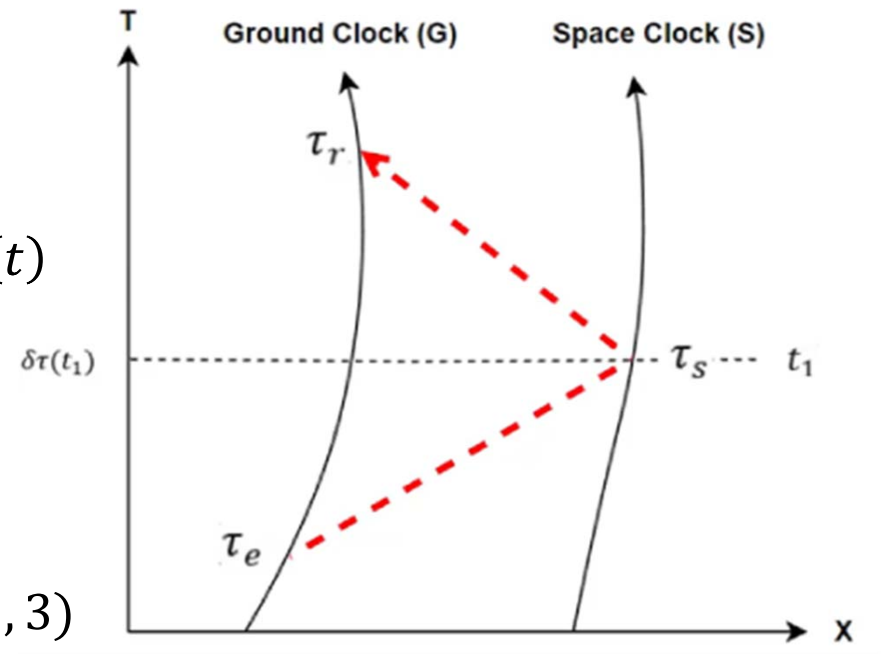
- Clock G generated laser pulses at proper emission time τ_e and transmits them toward space clock (S). Clock S then detects laser pulses at proper reception time τ_s and instantaneously reflects them back toward clock G, where they are received at the proper return time τ_r

- Time offset $\delta\tau(t)$ between clock S and G :

$$\delta\tau(t) = \frac{1}{2}(\tau_e + \tau_r) - \tau_s + \Delta\tau_{\text{cal}} + \sum_{i=1}^{3} \Delta\tau^i_{\text{delay}}(t) \quad (2)$$

$\Delta\tau_{\text{cal}}$: Calibration of all instrument delays

$\sum_{i=1}^{3} \Delta\tau^i_{\text{delay}}(t)$ are time delay order c^{-i} ($i = 1, 2, 3$)



Laser transmits at a wavelength of 532 nm with a repetition rate of **1 kHz**

2. Methodology

Analysis of Time Delays in TWLTT Link

$$\sum_{i=1}^{3} \Delta\tau_{\text{delay}}^{i}(t) = \Delta\tau_{GeM}^{1} + \Delta\tau_{Atm}^{1} + \Delta\tau_{Sag}^{2} + \Delta\tau_{Rel}^{2} + \Delta\tau_{Sag}^{3} + \Delta\tau_{Sh}^{3} + \Delta\tau_{\text{J2}}^{3} + \Delta\tau_{\text{noise}} \quad (3)$$

Where:

$\Delta\tau_{GeM}$: Geometric effect

$\Delta\tau_{Atm}$: Atmospheric correction

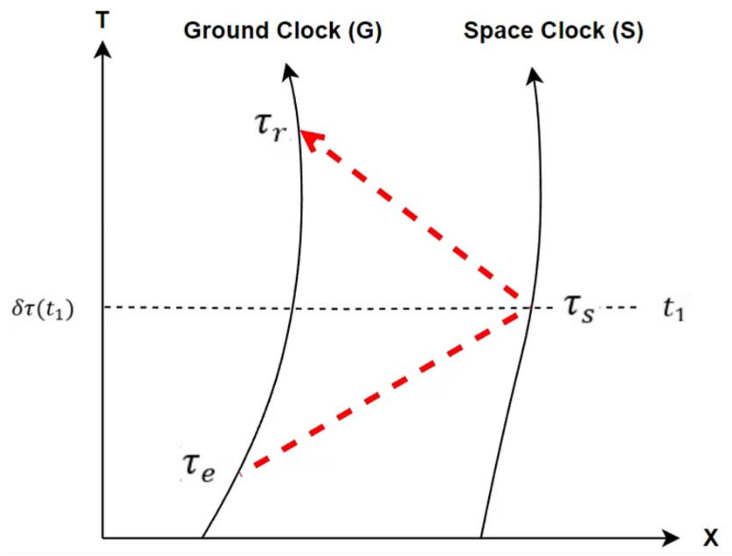
$\Delta\tau_{Sag}$: Sagnac delay

$\Delta\tau_{Rel}$: Relativistic frequency/Doppler effect

$\Delta\tau_{Sh}$: Shapiro delay

$\Delta\tau_{\text{J2}}$: Quadrupole moment term J2

$\Delta\tau_{\text{noise}}$: Drift in clocks, UTC offset,…etc.



- Due to the laser uplink and downlink being very close (~ 10 ms or 10 µs), only delays up to c^{-2} are considered in TWLTT Link. However, all delays are included in ranging computations.

2. Methodology

$$\boxed{\frac{\Delta f}{f}} = (1+\alpha)\boxed{\frac{\Delta U}{c^2}} \quad (1)$$

Measure Predicted

▪ Gravitational Redshift Test

According to General Relativity (GR), the proper time τ of a clock is given (Wolf and Petit, 1995):

$$\frac{d\tau}{dt} = 1 - \left(\frac{U}{c^2} + \frac{v^2}{2c^2}\right) + O(c^{-4}) \quad (4)$$

Integrating with respect to coordinated time can be used to compare clocks G and S in the time domain as (Ruby et al., 2024) :

$$\underbrace{\Delta\tau(t) = \boldsymbol{\tau^G(t) - \tau^S(t)}}_{\text{TWLTT Link}} = \Delta\tau_0 - \int_{t_0}^{t} \frac{(U_G - U_S)}{c^2} dt - \int_{t_0}^{t} \frac{(v_G^2 - v_S^2)}{2c^2} dt \quad (5)$$

In our study, $$\boldsymbol{\Delta\tau^m = \Delta\tau_0 + (1+\alpha)\,\Delta\tau^{GR}} \quad (6)$$

Measure value

$$\boxed{\Delta\tau^m = \Delta\tau(t) + \int_{t_0}^{t} \frac{(v_G^2 - v_S^2)}{2c^2}\, dt}$$

Predicted value from GR Model

$$\boxed{\Delta\tau^{GR} = -\int_{t_0}^{t} \frac{(U_G - U_S)}{c^2} dt}$$

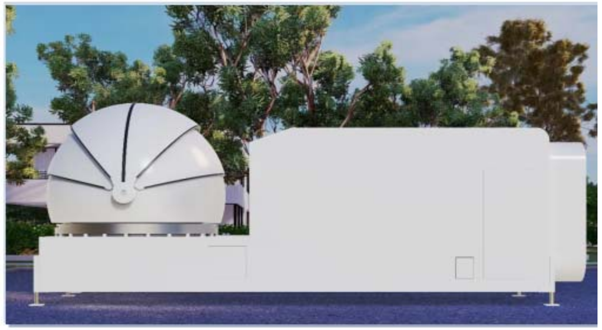
3. Testing Gravitational Redshift with LTE

▪ Experimental Setup

- We Choose time and frequency station at Xi'an as ground laser station.
- Experiment data in October 2024 covers a **3-day** period from Modified Julian Date (MJD) 60584 to 60586 (**6 passes/day**, each lasting about **6 minutes**).

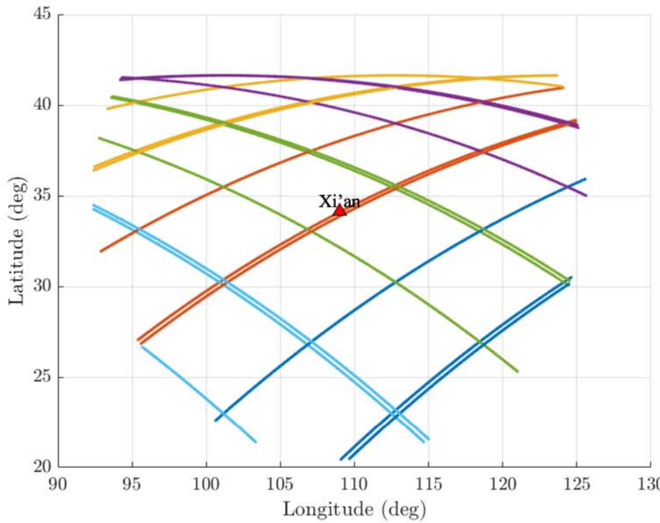
Table (2): Parameters of Simulation and TWLTT link.

Items	Values of Parameter
Mission Name	CSS (TIANHE)
Altitude	400 ~ 450 Km
Orbit inclination	42° ~ 43°
Orbit data	TLE file from China Manned Space Flight: https://www.cmse.gov.cn/
CSS orbit interval	1.0 s
Position/ Velocity accuracy of CSS	0.1 m and $1 \times 10^{-5}\ m/s$
Observation cutoff elevation angle	5° ~ 85°
On-board clock stability	$2 \times 10^{-15}/\sqrt{\tau}$
Ground clock stability	$1 \times 10^{-15}/\sqrt{\tau}$
Meteorological data	An ERA5-Based Hourly Global Pressure and Temperature (HGPT) Model
Laser wavelength	532 nm
Pulse Energy	~1.5 mJ
Pulse width	30 ps
Repetition rate	1 kHz
Gravity field model	EGM2008 for Relativistic delay & term $\Delta\tau^{GR}$
Tropospheric model	Mendes and Pavlis (2004)
Time delay function	Blanchet et al. (2000)
Tide correction	ETERNA



Ground Laser Station at Xi'an

Lat. (deg)	Lon. (deg)	h (m)
34.142511	108.99657	569.247

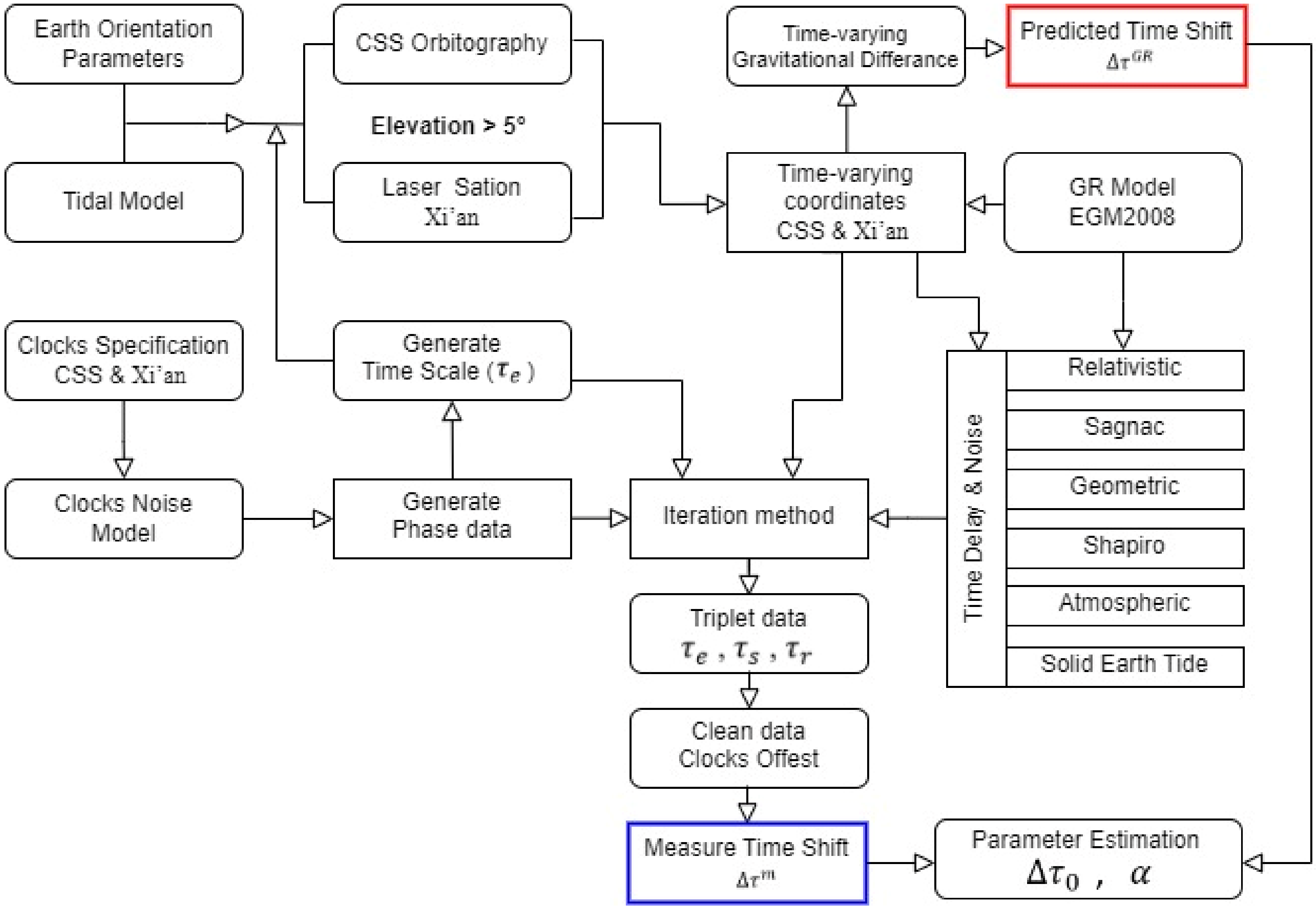


Orbit of CSS during observations

3. Testing Gravitational Redshift with LTE

▪ Experimental Setup

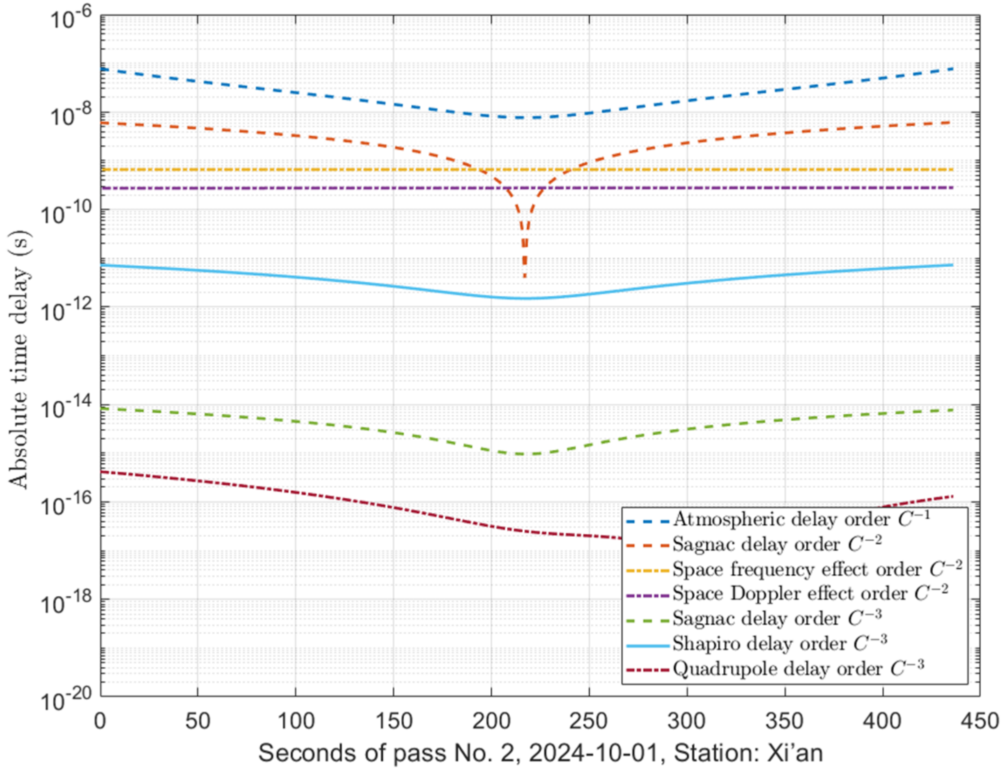
Scheme of Simulation Experiment



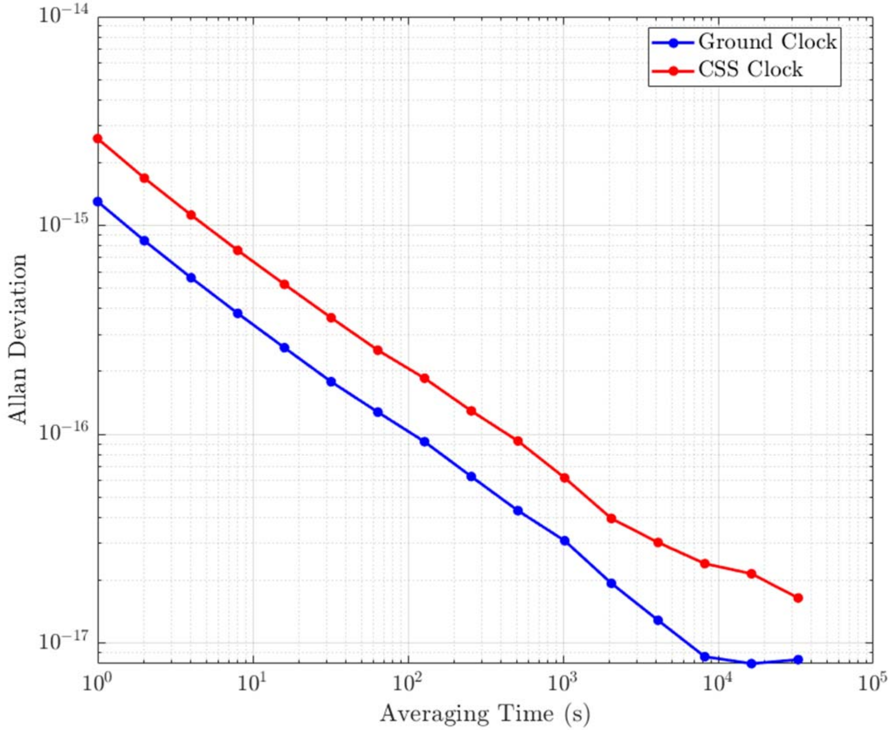
3. Testing Gravitational Redshift with LTE

▪ Results and Analysis

Absolute Time Delays and Clocks Stability



- Different time delay simulation in a one-way laser time link.



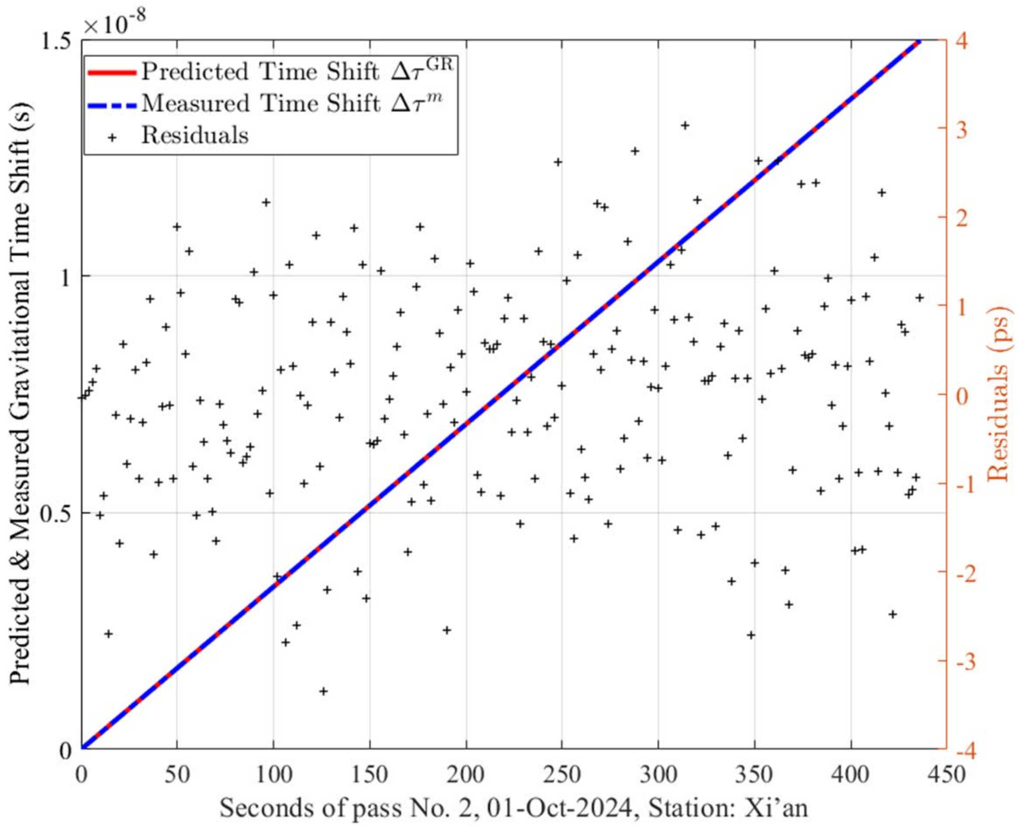
- Simulation clocks data for CSS and Xi'an ground clock.

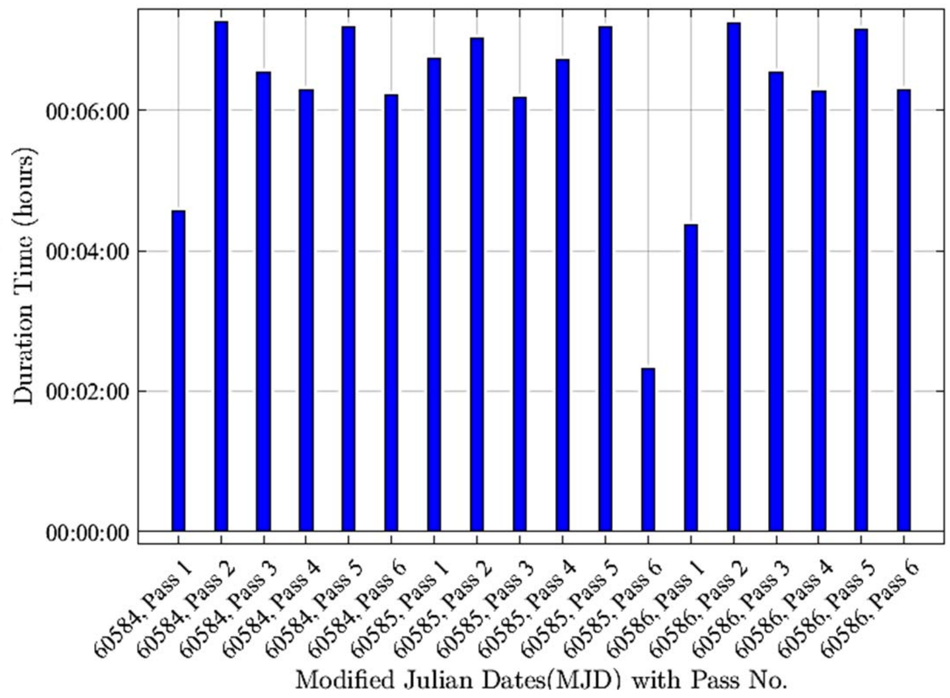
- In TWLTT link, only delays up to order (c^{-2}) remain due to laser uplink/downlink **very close.**
- Other factors, such as ground and spacecraft system delays, geometric corrections, and Earth's tidal effects, significantly influence the results (Geng et al., 2023), they are not considered in this simulation.

3. Testing Gravitational Redshift with LTE

- **Results and Analysis**

Predicted and Measured Gravitational Time Shifts Over One pass (simulated)





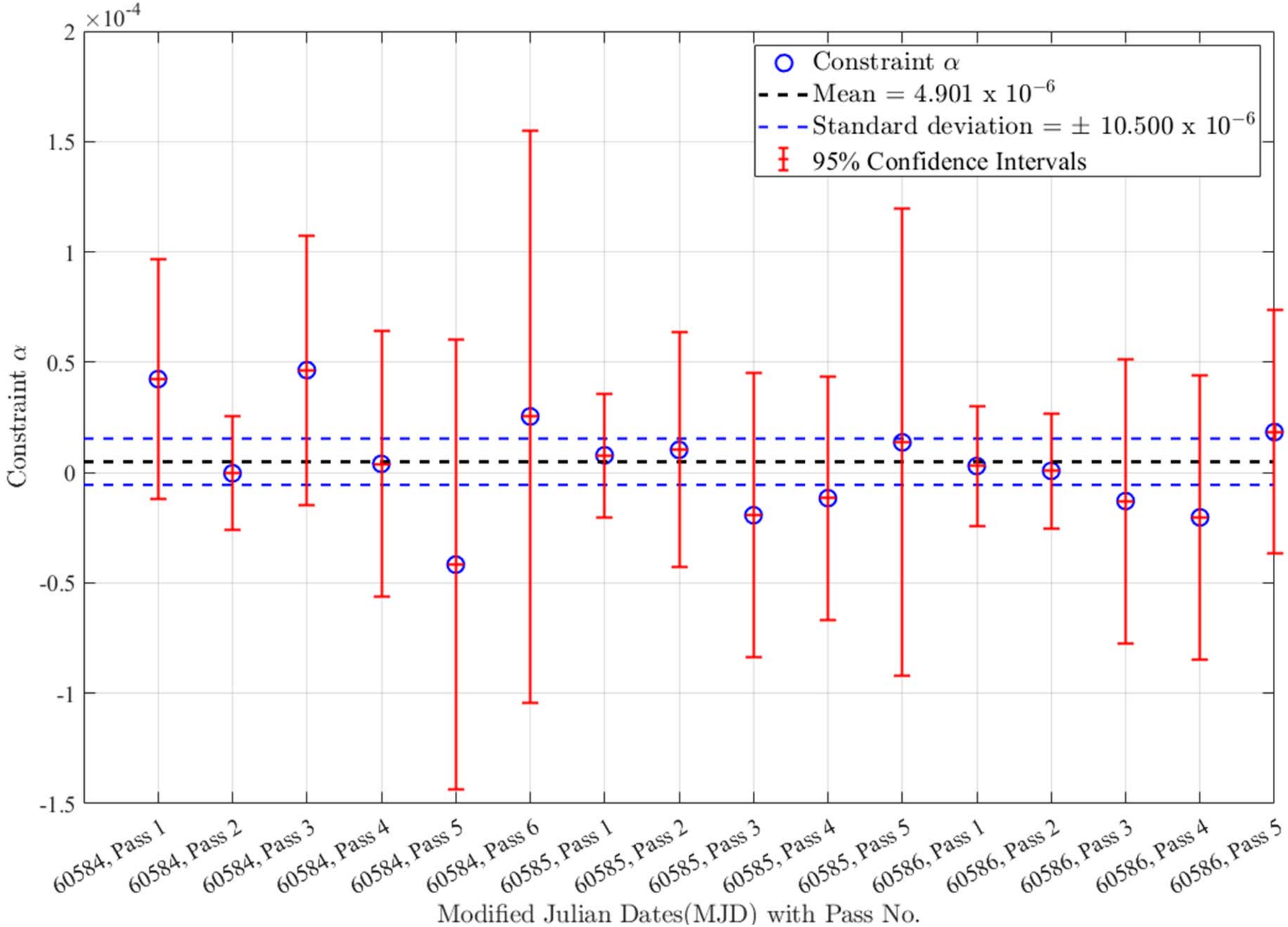
Using single pass (Length ≈ 7 minutes), accuracy of test **GRS**

$$\alpha \approx (0.324 \pm 13.13) \times 10^{-6}$$

3. Testing Gravitational Redshift with LTE

▪ Results and Analysis

Simulated Gravitational Time Shifts Results at Xi'an Laser Station



4. Conclusion

- The experiment utilizes state-of-the-art optical atomic clocks with exceptional stabilities ($\sim 3 \times 10^{-17}$ @ day) and a novel single-photon detector with superior timing stability.
- In two-way laser time transfer (TWLTT) method, only delays up to order (c^{-2}) remain.
- The optical TWLTT method achieves an accuracy of about $\mathbf{1.0 \times 10^{-5}}$ for testing gravitational redshift (GRS), as confirmed by short-period simulated observations in Laser Timing Experiment (LTE).
- Testing GRS is currently in progress using real data from the LTE aboard the CSS, aiming to achieve an accuracy level of 10^{-7}.
- **Challenges** : Calibration of ground and space equipment.

Thanks for your Attention

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