中科院国家天文台长春人造卫星观测站
Changchun Observatory, NAOC, Chinese Academy Of Sciences
23RD INTERNATIONAL WORKSHOP ON LASER RANGING (IWLR)
Oct.20~26, 2024
Kunming, China

Rapid Solution of Earth Rotation Parameters by LLR Common View: A Numerical Simulation

LIANG Zhi-Peng*, LIU Cheng-Zhi*, DONG Xue
Email: liangzp@cho.ac.cn
Changchun Observatory of National Astronomical Observatories, Chinese Academy of Sciences,
Jingyuetan Lake Xi-Shan 1, Changchun, 130117, China

Abstract

Lunar laser ranging (LLR) has been used to solve for earth rotation parameter (ERP) since 1970s. Usually, the solutions incorporate state-of-the-art models and use as much LLR data as possible, spreaded during decades. With the current infrastructure of LLR station network and modern measurement accuracy, it is now possible to envisage rapid solution of ERPs in a common-view observation. Numerical simulations on tri-static common-view scenarios were analysed, to explore the stiffness of the problem. The rapid solution has potentiality to address short-term and transient geophysical changes. However, the method relies on long-term geodetic products, and therefore can not replace conventional solutions.

Introduction

Determining the Earth's orientation in inertial space finally leads to the so called Earth Rotation Parameters (ERPs). The determination of ERPs traditionally requires long observation period, resulting in significant delays. Solution of ERP had been tried since first LLR data (Bender et al. 1973). Leick (1980) did theoretical research on LLR common view to measure earth orientation in short observation period. In this work, we explore the possibility of LLR common view in selected laser ranging sites with numerical simulation. Our simulation is based on modern geodetic theory and technical conventions (Petit et al. 2010).

Earth Rotation Parameters

Conventionally, earth rotation parameter (ERP) contains three numbers. Two of them are terrestrial intermediate pole coordinates known as xp, yp. The third one is the deviation of earth rotation from steady rotator, traditionally quantitized in UT1-UTC along with leap seconds, provided by IERS bulletins. In this work we adopt UT1-TAI as the third ERP, and quantize its variation in arcseconds of earth rotation angle. For better formulation, we reorder them as (P_1, P_2, P_3) = (y_p, x_p, ERA), and their variations (p_1, p_2, p_3)=(Δy_p, Δx_p, ΔERA). Such order corresponds to rotation transformations around x, y and z axes of terrestrial frame.

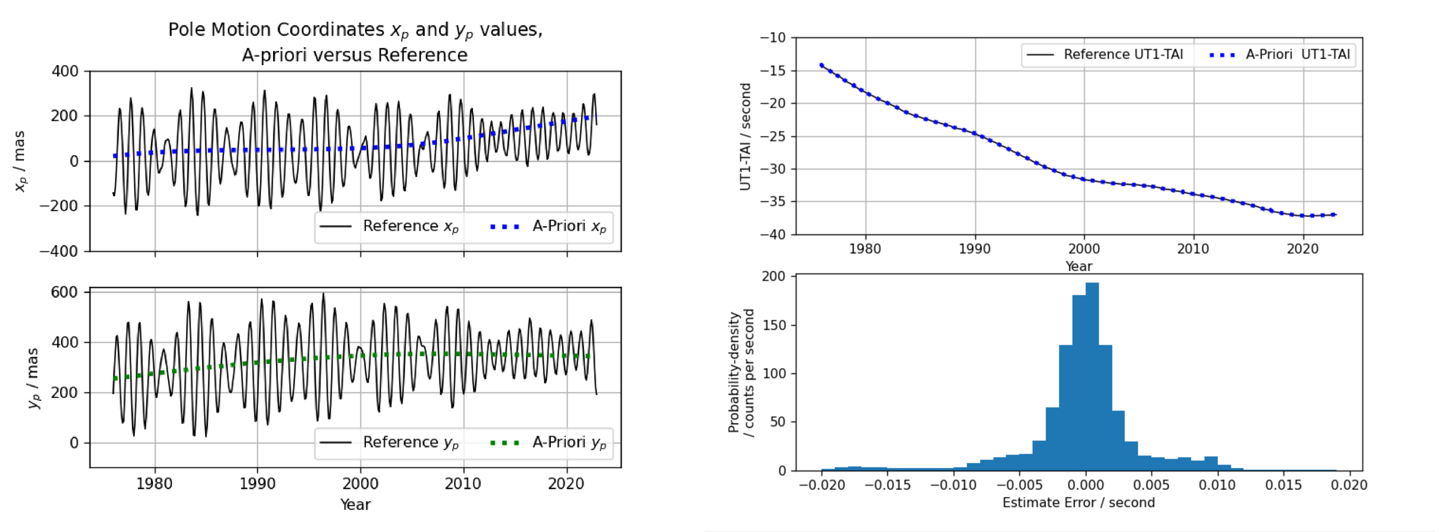


Figure 1 Earth Rotation Parameters x_p, y_p and UT1-TAI, Reference Values vs. Approximation Values.

Observatory Networks

We found some tristatic combinations (networks or nets) over a selected subset of laser ranging stations that exist in China, Europe, and Southern hemisphere. The APOL is not included in this work, due to the lack of common view opportunity with other selected sites. The networks are listed in Table 1 and marked in Figure 2 as colored triangles.

Among them the ▲EUR network is densest and has the highest latitude. The ▲EA2 is largest in size and its center is near equator. Around the KUNL site there are supple choices to form common view network, of which we picked four. The ▲CHN network is located inside China. The ▲CP2 network incorporates both high and low latitude and southern hemisphere sites. The ▲CS1 network has largest longitude span (i.e. time span). The ▲CP1 managed to include high latitude sites, with two in northern and one in southern hemisphere. One can discover some insights about forming common view network, e.g., lower latitude gives more opportunity, smaller triangle is easier, and longitude span should not exceed 8 hours.

In this work, lunar common view means in all member sites, the moon elevation angle > 20°, and sun-moon separation angle > 30°.

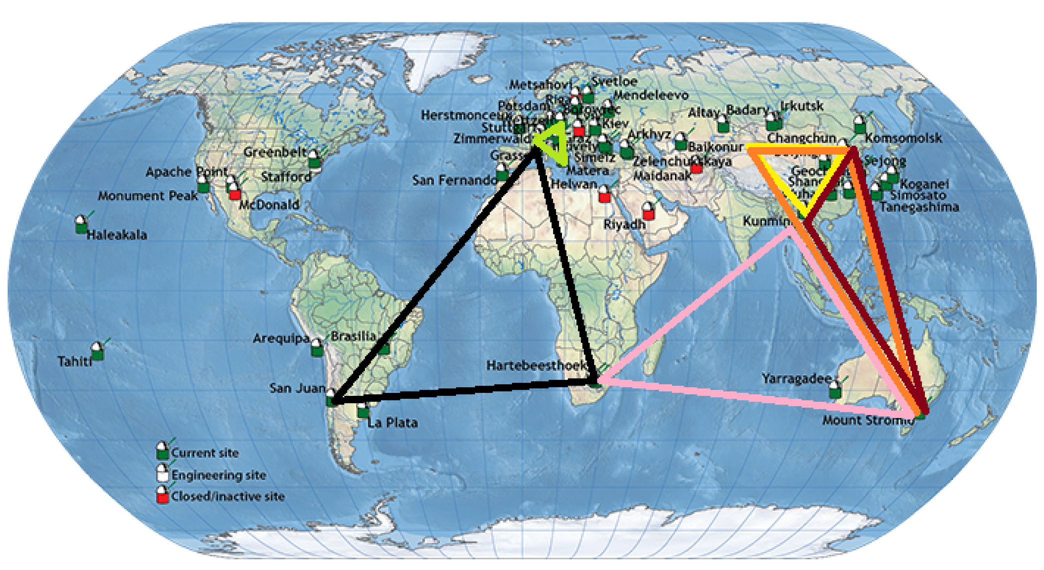


Figure 2. Tristatic Networks. Each network is marked with a colored triangle.

Table 1. Tristatic Networks

Net Name	Stations
▲EUR	GRSM, MATM, WETL
▲CHN	CHAL, KUNL, URUL
▲CP2	CHAL, KUNL, STL3
▲CP1	CHAL, URUL, STL3
▲EA2	GRSM, HARL, SJUL
▲CS1	KUNL, HARL, STL3

Simulation Data

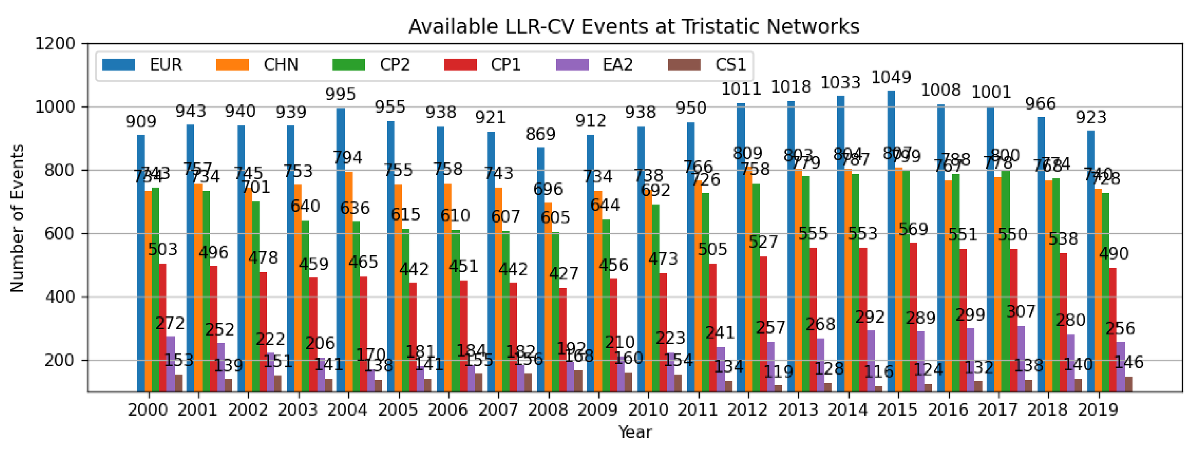
For each observatory, sun and moon angles are calculated at the beginning of each hour, for years 2000-2019. Common view events are found by enumerating over hour grids, with the conditions (moon elevation angle > 20°, and sun-moon separation angle > 30°) to be satisfied at all member sites.

Partial derivative matrix (design matrix) were calculated for networks at each hour grid. Columns of design matrix correspond to ERP variations (p_1, p_2, p_3) and rows correspond to different observatories. Each element of design matrix is a partial derivative of range measurement to either one of ERP variation.

One may solve the equation directly, for all three ERPs (the '3-by-3-problem'), or form over determined sub-problem to solve for one or two ERPs('3-by-1- and 3-by-2-problem'), using least square method.

In this work, only stiffness analysis was done. See next section.

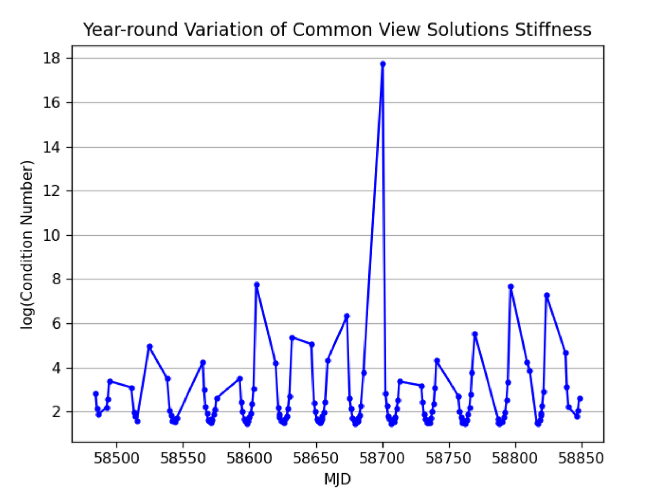
Figure 3. Common view events available for tristatic networks, in each year. The EUR has the most common view events, because its member sites are close to each other. Network CS1 has the least, due to its large span in longitude, making it difficult to meet common view conditions.



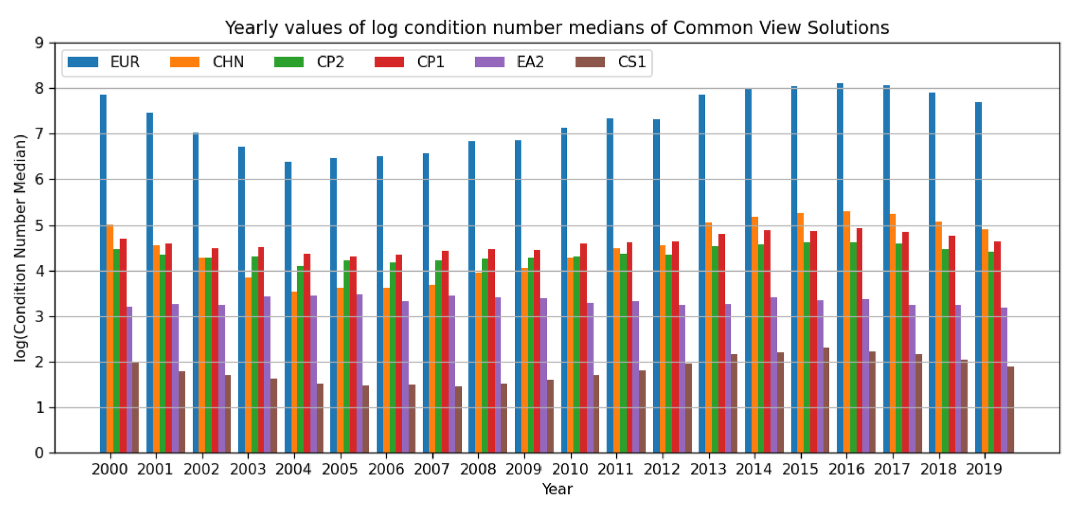
Stiffness Analysis

For the common view problem in a given tristatic network, the stiffness only relies on the condition number of design matrix. Calculation on the simulated data showed extremly high condition numbers for the 3-by-3-problems. This means although observation networks are triangular, the moon is too far to form a well defined problem for all three ERPs.

For 3-by-2-problems, here we assume accurate value of UT1 or ERA, and solve for polar motion coordinates xp and yp. The 3-by-2-problem then converts to a 2-by-2 normal equation. Analysis on the 2-by-2 normal matrix produces Figure 4 and Figure 5. The condition number or stiffness varies during every synodic month, but the magnitude is determined by network.



↑ Figure 4. Variation of condition number of CHN common view problem during year 2000. Note y-axis is logarithm. It is easy to notice synodic period in the data. In particular days during a synodic month, the condition number drops to below 100, then worsen to infinity in other time.



← Figure 5. Logarithm of condition number medians, of six networks and years 2000-2019, categorized by network and year. Note y-axis is logarithm. The EUR network has highest stiffness due to spatial density. Network CS1 and EA2 are less stiff due to large spatial span.

Conclusion & Discussion

Solving for all ERPs in one common-view LLR observation, is not realistic, due to the stiffness in the equation. The moon is so far from the earth, that tri-static common-view range measurement to lunar target is essentially 2-D measurement. But if UT1 is determined ahead, it is possible to solve for polar motion coordinates with tri-static common-view measurements.

Reference

Bender, P. L., D. G. Currie, R. H. Dicke, et al. (1973). "The Lunar Laser Ranging Experiment." Science **182**(4109): 229-238.
Leick, A. (1980). "Potentialities of Lunar laser range-differencing for measuring the Earth's orientation." Bulletin Géodésique **54**(1): 55-68.
Petit, G. and B. Luzum (2010). IERS Conventions (2010). Frankfurt am Main, Verlag des Bundesamts für Kartographie und Geodäsie.
(and more...)

TWENTY THIRD INTERNATIONAL WORKSHOP ON LASER RANGING KUNMING 2024 OCTOBER 21-25