Improving multiple LEO combination for SLR-based geodetic parameters determination using variance component estimation

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Outline

We focused on improving the integration of SLR observations based on **VCE**.

Introduction

- The combination of multi-LEOs SLR
- The application of VCE in SLR
- Motivation

Methodology

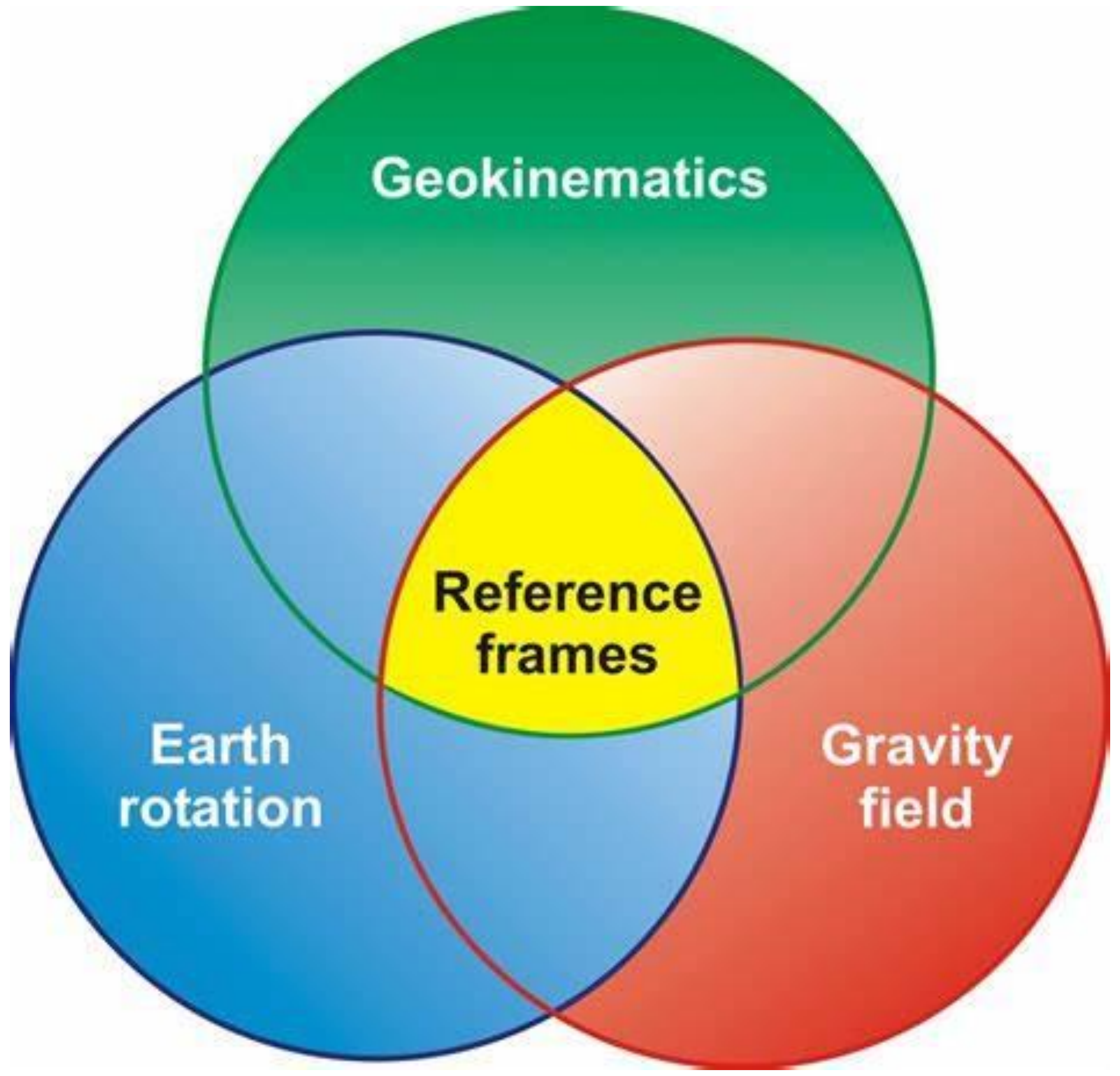
Result

- SLR solution under different covariance matrices
- SLR solution under monthly weights

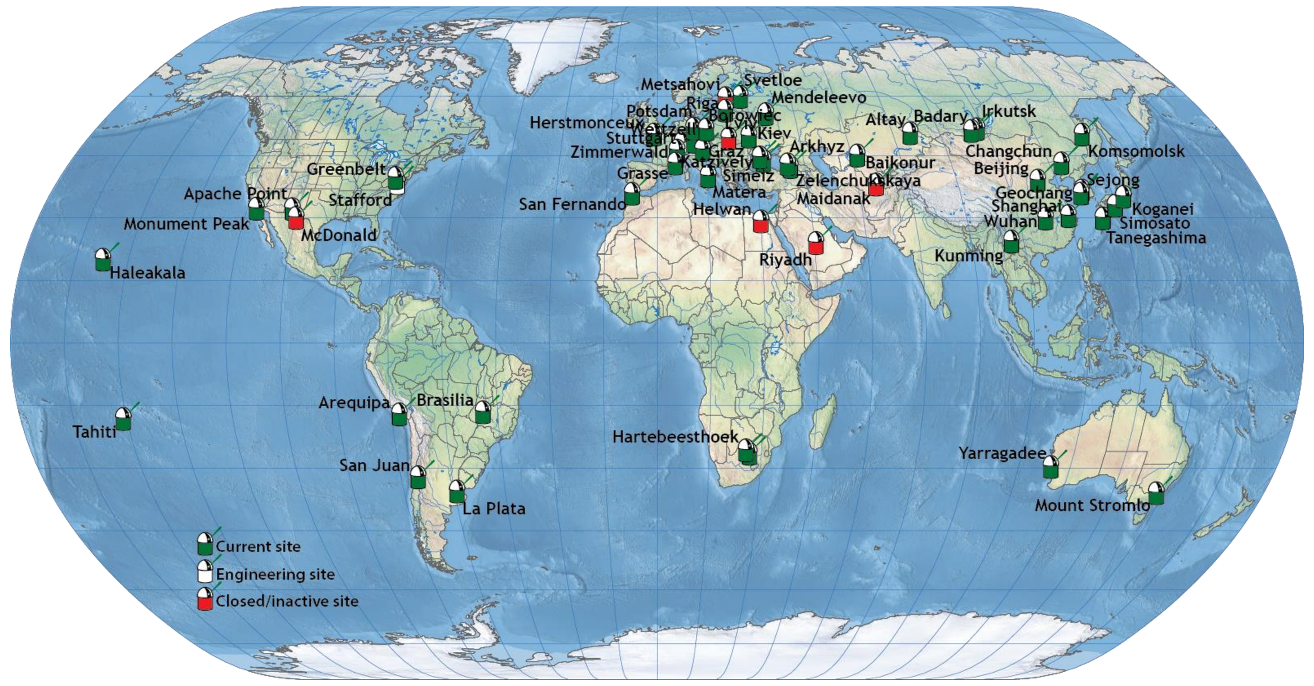
Conclusion

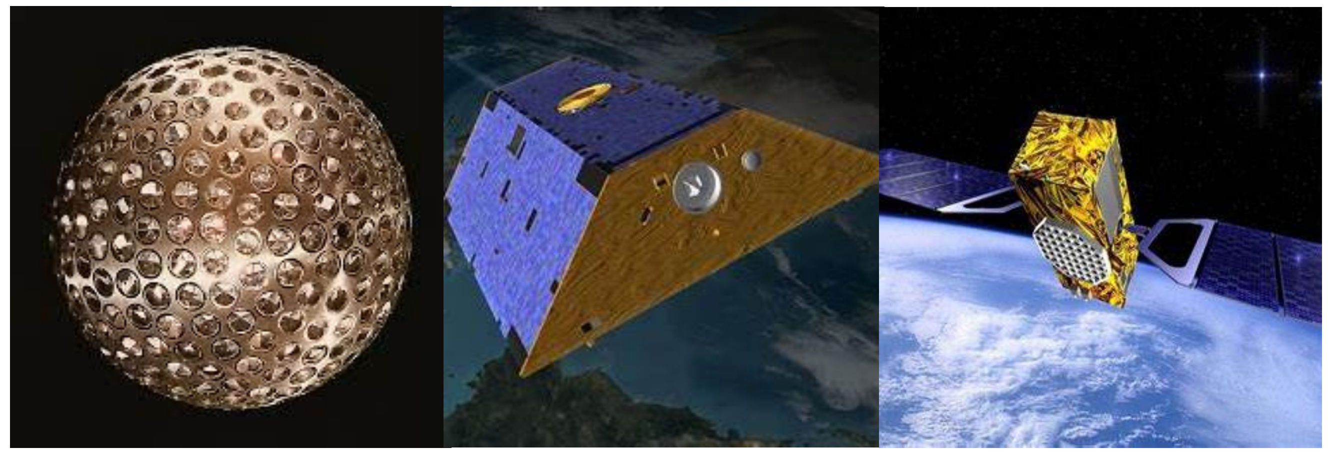
The significance of SLR

Provides accurate
measurements of the distance



Integrates the three major pillars
of the Geodesy

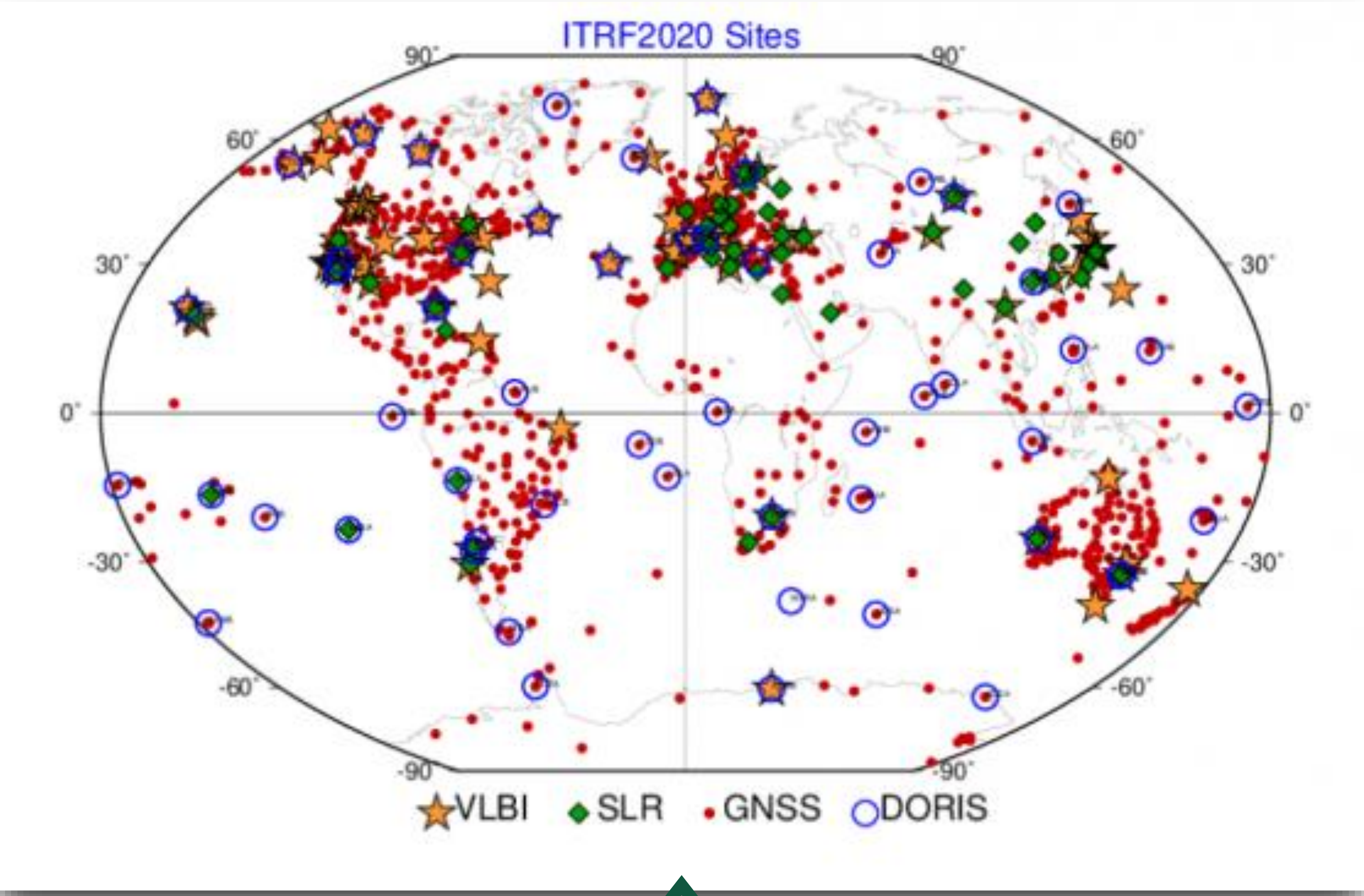


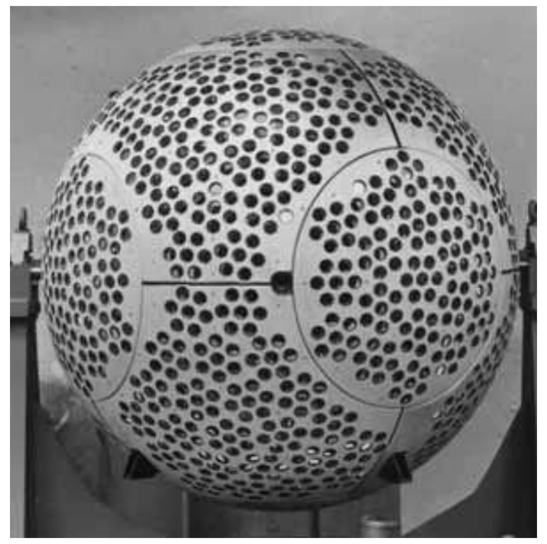


Station: over 40
Satellites: LEO HEO LAGEOS

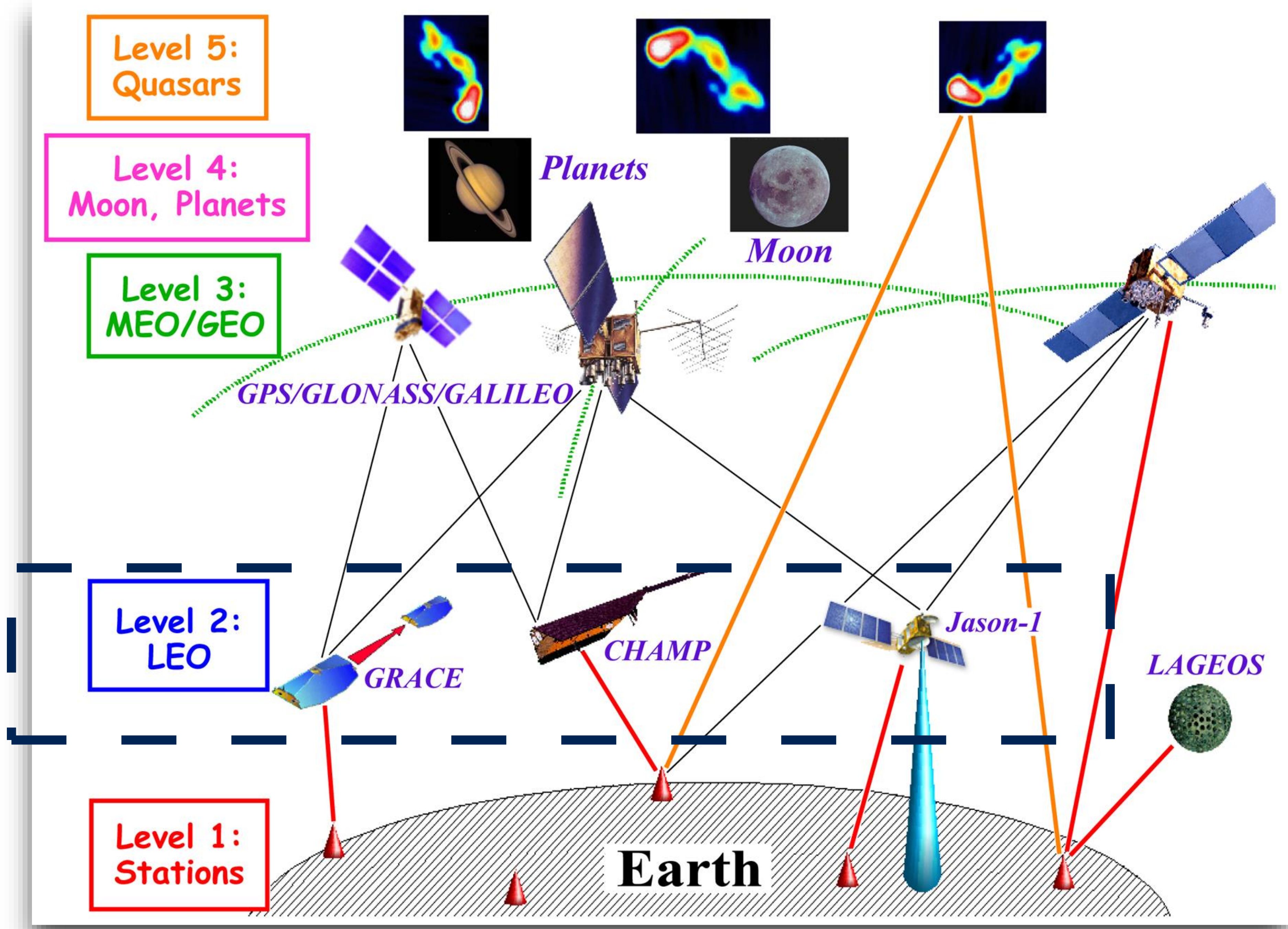
The combination of multi-LEOs SLR

- Only the observations of **two LAGEOS** satellites and **two Etalons** contributes to ITRF2020





- Goal of **0.1 mm**/year station velocities and **1mm** coordinates accuracy



Integrate the SLR observations from **multiple LEO** satellites to improve the accuracy and stability of SLR-derived parameters

Motivation

- **distribution** of stations
- **instrument** of stations
- **operation** of stations
- **orbit** determination of satellites
- **instrument** of satellites

→ Quality inhomogeneity in SLR observations (Sahin et al. 1992; Strugrek et al. 2021)

↓ Realistic stochastic model for multi-LEO SLR observations

VCE is a classic method used to obtain the variance component of the **heterogenous observation** types and find the realistic and reliable weight of the observation.

Question1: SLR solution under different covariance matrices

VCE

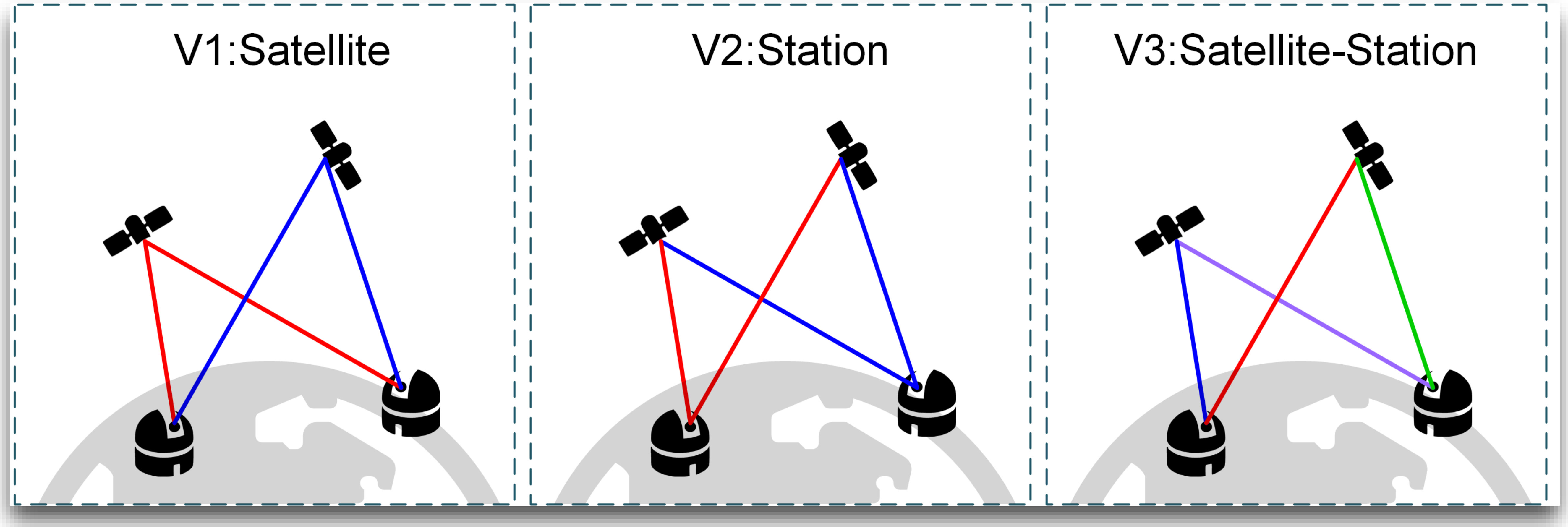
$$\sigma_A^2 = \frac{V_A^T P_A V_A}{r_A}$$

$$\sigma_B^2 = \frac{V_B^T P_B V_B}{r_B}$$

How to define the variance component A and B ?

The **structure** of the covariance matrix

$$\mathbf{Q}_y = \mathbf{Q}_0 + \sum_{k=1}^{p} \sigma_k^2 \mathbf{Q}_k$$



The difference in quality

V1: Satellite

$$\mathbf{Q}_y = \begin{pmatrix} \sigma_{Sat_1}^2 \mathbf{Q}_{Sat_1} & & & \\ & \sigma_{Sat_2}^2 \mathbf{Q}_{Sat_2} & & \\ & & \ddots & \\ & & & \sigma_{Sat_h}^2 \mathbf{Q}_{Sat_h} \end{pmatrix}$$

V2: Station

$$\mathbf{Q}_y = \begin{pmatrix} \sigma_{Sta_1}^2 \mathbf{Q}_{Sta_1} & & & \\ & \sigma_{Sta_2}^2 \mathbf{Q}_{Sta_2} & & \\ & & \ddots & \\ & & & \sigma_{Sta_q}^2 \mathbf{Q}_{Sta_q} \end{pmatrix}$$

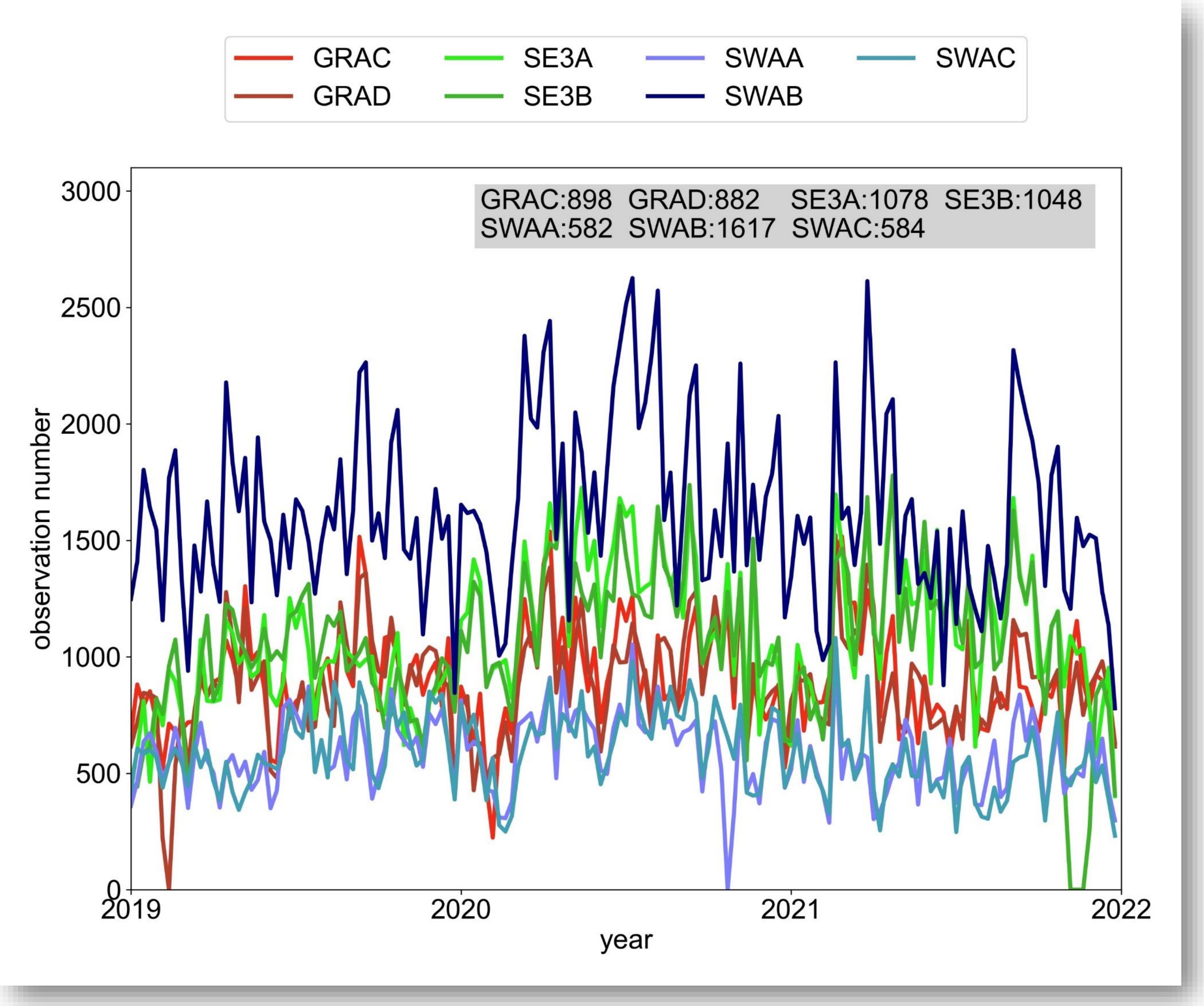
V3: Satellite-station

$$\mathbf{Q}_y = \begin{pmatrix} \sigma_{Sat_1\_Sta_1}^2 \mathbf{Q}_{Sat_1\_Sta_1} & & & \\ & \sigma_{Sat_1\_Sta_2}^2 \mathbf{Q}_{Sat_1\_Sta_2} & & \\ & & \ddots & \\ & & & \sigma_{Sat_h\_Sta_q}^2 \mathbf{Q}_{Sat_h\_Sta_q} \end{pmatrix}$$

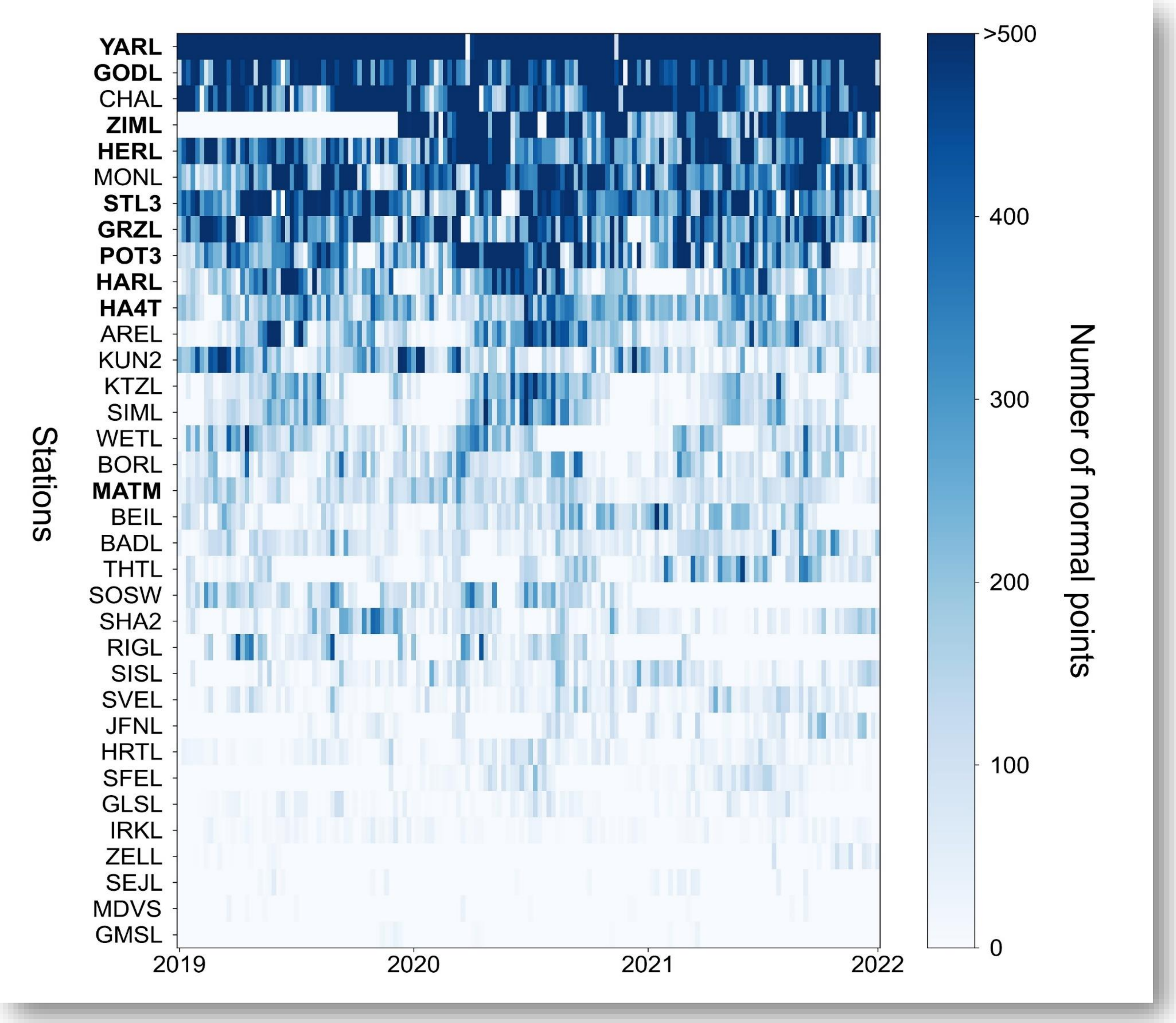
Question2: SLR solution under monthly weights

According to Zajdel et al. (2019), 50 normal points per week are sufficient to calculate the coordinates with residuals lower than 10 mm.

Satellite



Station



For some non-core station, there is obvious insufficiency in the observation for VCE.

VCE and its simplified method

Equations system: $$E(y) = Ax, D(y) = Q_y = E\{(y - Ax)(y - Ax)^T\} = Q_0 + \sum_{k=1}^{p} \sigma_k Q_k$$

LS estimator: $$\hat{\boldsymbol{\sigma}}^2 = \mathrm{N}^{-1}\mathrm{l}$$

Component of N: $$\mathrm{n_{ij}} = \frac{1}{2}\mathrm{tr}\left(\mathrm{Q_i W_y P_A^{\perp} Q_j W_y P_A^{\perp}}\right) i, j = 1 \ldots p$$

Component of l: $$\mathrm{l_i} = \frac{1}{2}\mathrm{\hat{v}^T W_y Q_p W_y \hat{v} i = 1 \ldots p}$$

The estimation needs iterations. Each iteration requires the repeated multiplication and inversions of **6000*6000** matrices, which is so **time-consuming**.

$$Q_y = \begin{pmatrix} \sigma_1^2\mathbf{Q}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \sigma_2^2\mathbf{Q}_2 & \cdots & \mathbf{0} \\ \vdots & & \ddots & \\ \mathbf{0} & \mathbf{0} & & \sigma_p^2\mathbf{Q}_p \end{pmatrix}$$

simplify

$$\mathrm{n_{ij}} = \delta_{\mathrm{ij}}\left(\mathrm{\bar{n}_i} - 2\mathrm{tr}\left(\mathrm{\bar{N}^{-1}\, A_i^T W_i A_i}\right)\right) + \mathrm{tr}\left(\mathrm{\bar{N}^{-1}\, A_i^T W_i A_i \bar{N}^{-1}\, A_j^T W_j A_j}\right)$$

$$\mathrm{l_i} = \mathrm{\hat{v}_i^T W_i \hat{v}_i}$$

stochastic model is block diagnonal structure

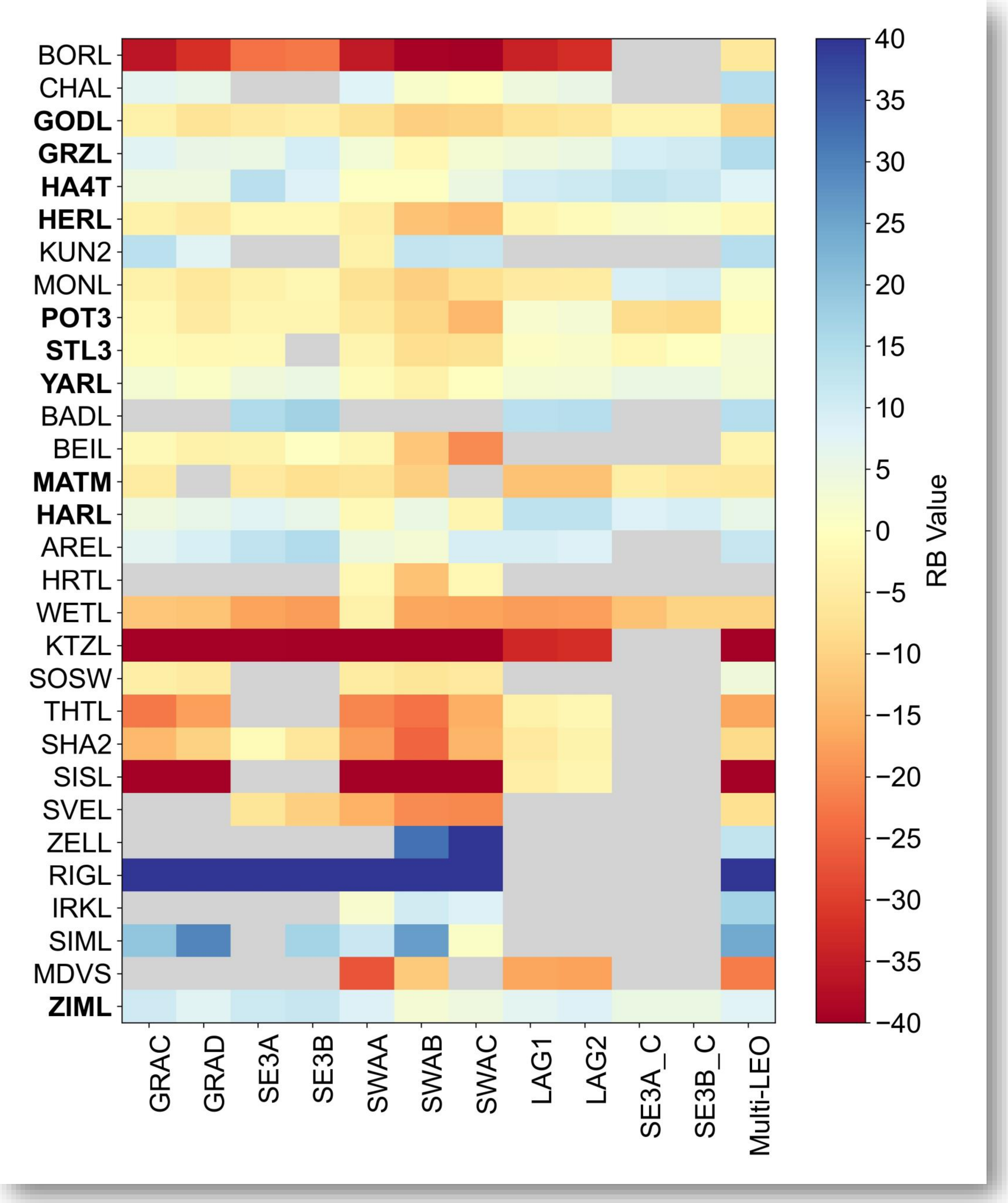
The simplification reduces the time of each iteration to seconds

Strategy

Satellites	GRACE-C GRACE-D Sentinel-3A Sentinel-3B Swarm-A Swarm-B Swarm-C
Time	2019-2021
Arc	7-day

Scheme	Description
V0	equally weighed
V1	satellites
V2	station
V3	satellite-station pair

- Rangebias: correct with the **long-term** solution



SLR solution under different covariance matrices : observations number

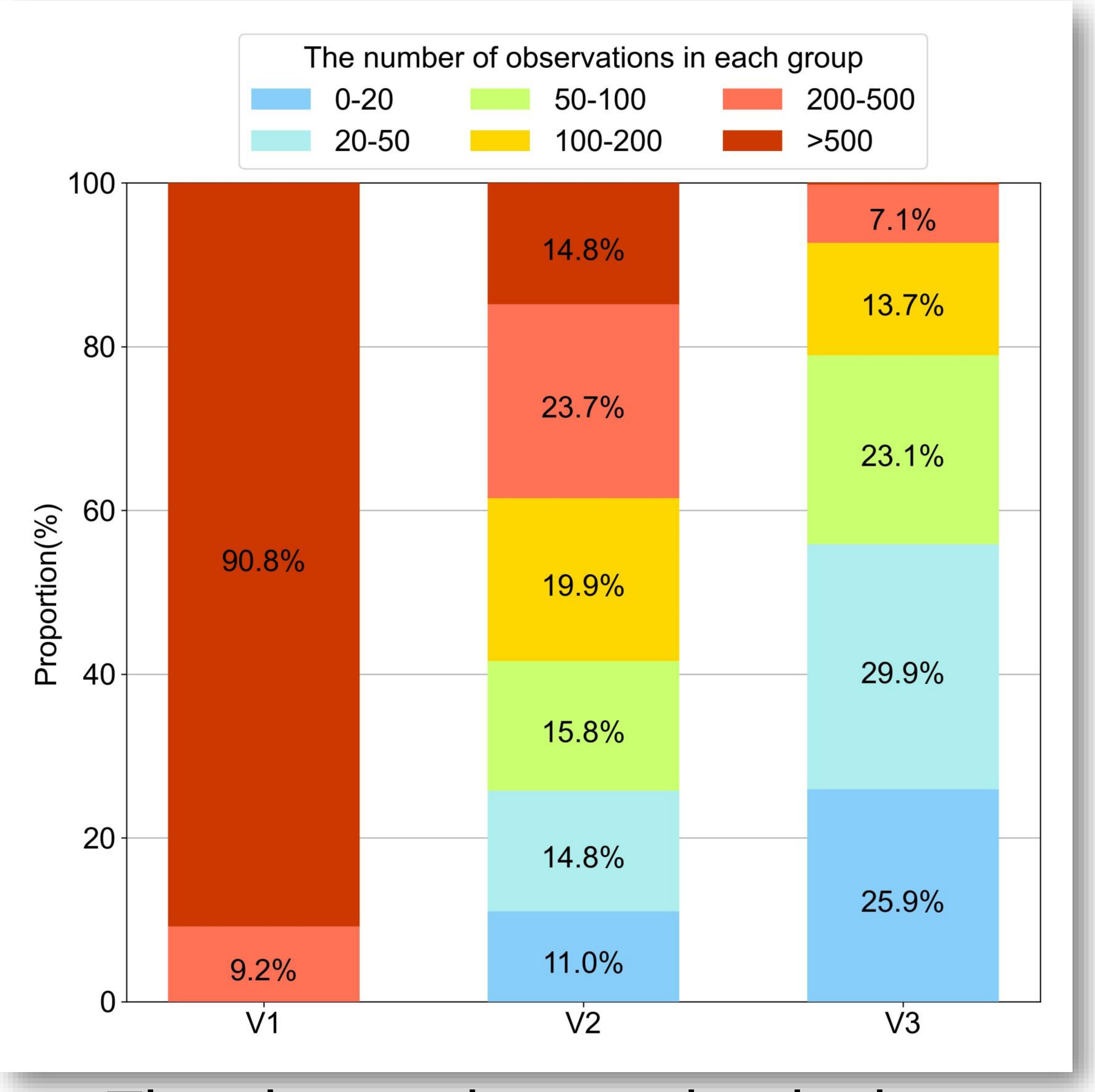


Figure: The observation number in the group under different schemes

- The division schemes have a direct impact on the quantity of the observation number in one group
- There are some groups with **insufficient** observations under V3 and V2 schemes

V0: All equal	V1: satellites	V2:stations	V3:satellite-station

SLR solution under different covariance matrices : Derived weight

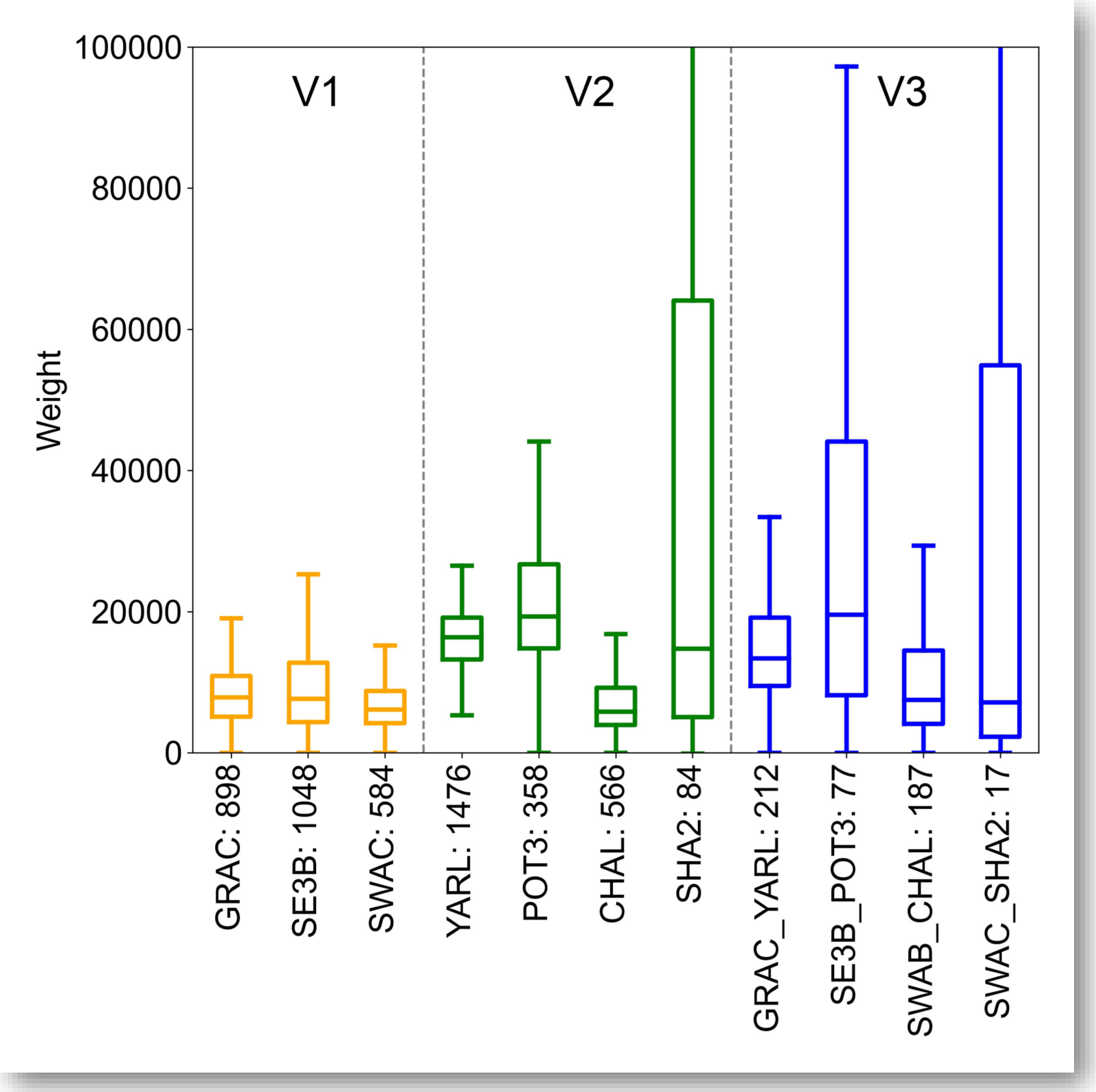


Figure: The boxplot of derived weight under different schemes

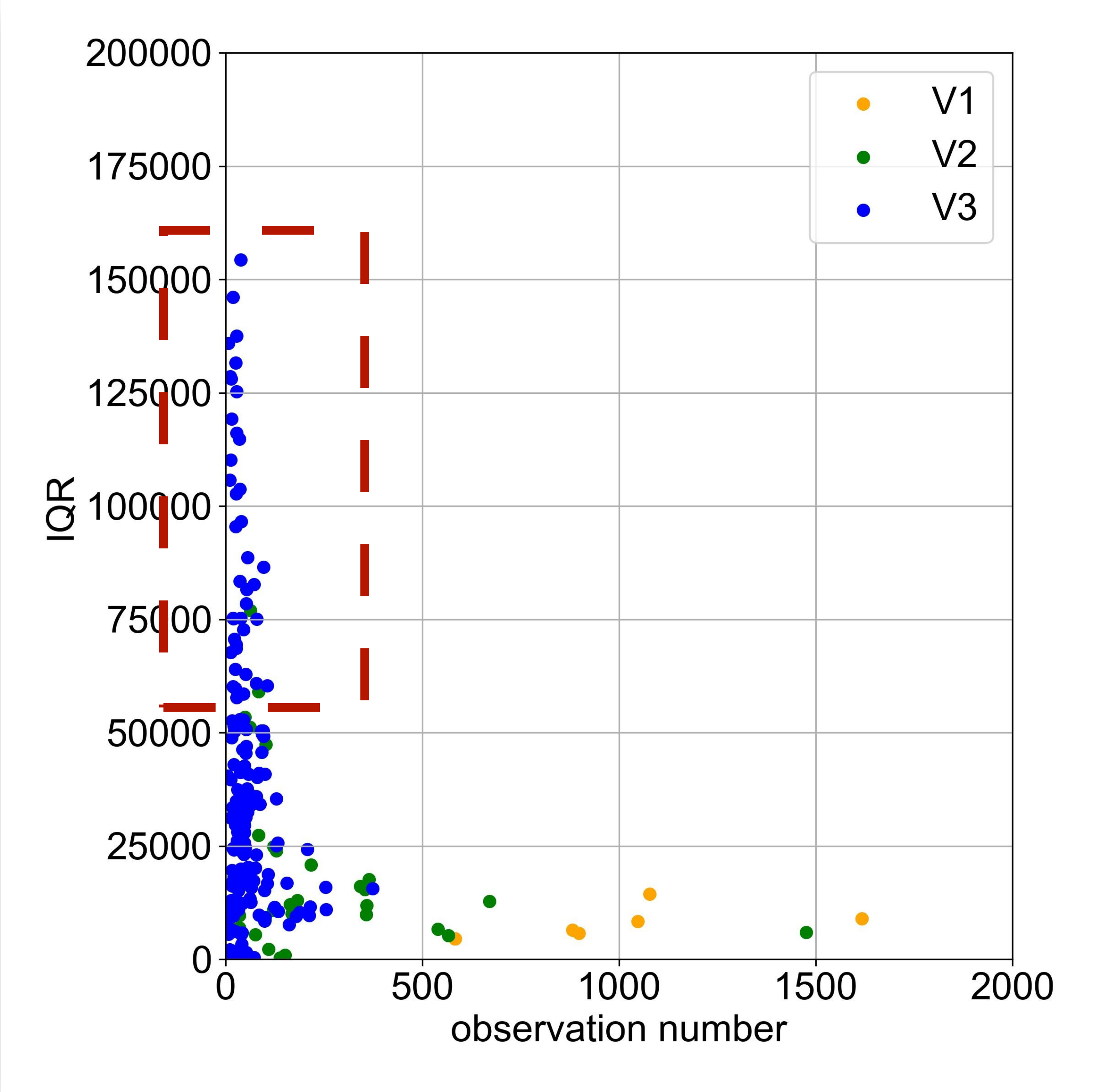
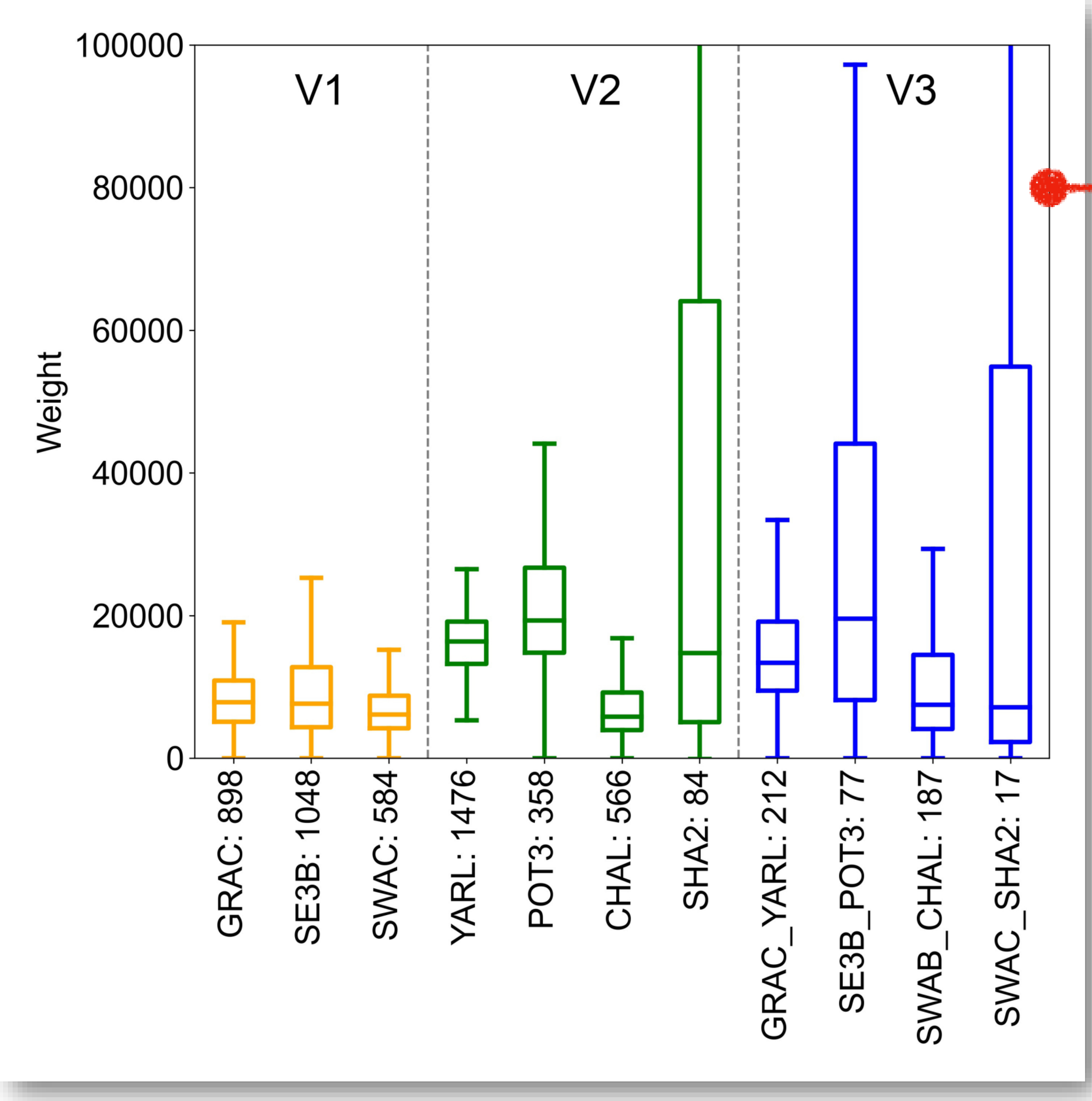


Figure: The IQR of weight and the its observation number in group under V3

- The IQR reflects the **fluctuate** of the variance components
- Groups with insufficient observations get the **far larger IQR** under V3
- Large IQR **only appears** within the insufficient observations in a group

V0: All equal	V1: satellites	V2:stations	V3:satellite-station

SLR solution under different covariance matrices : Derived weight



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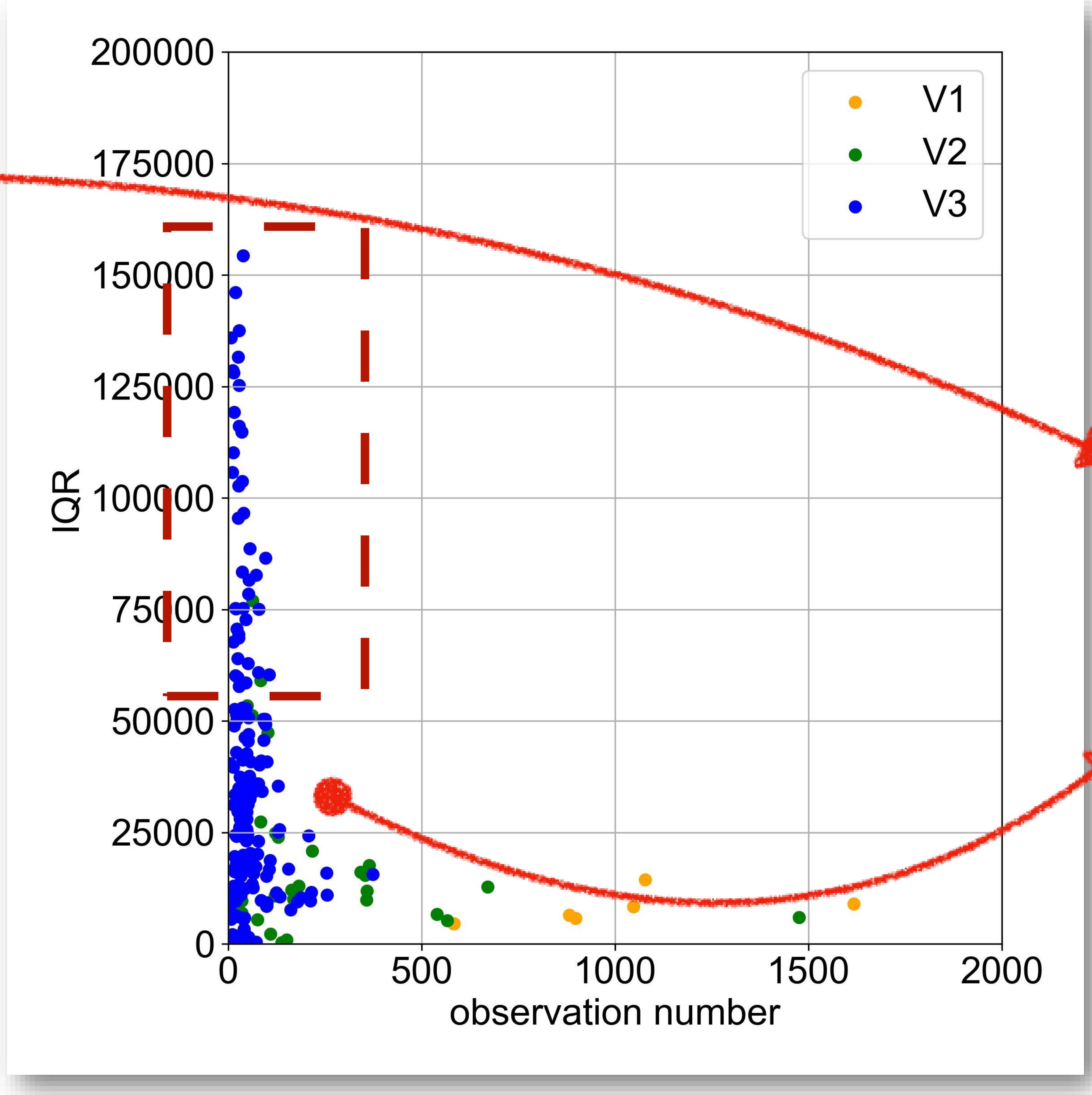

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SLR solution under different covariance matrices : Station coordinates

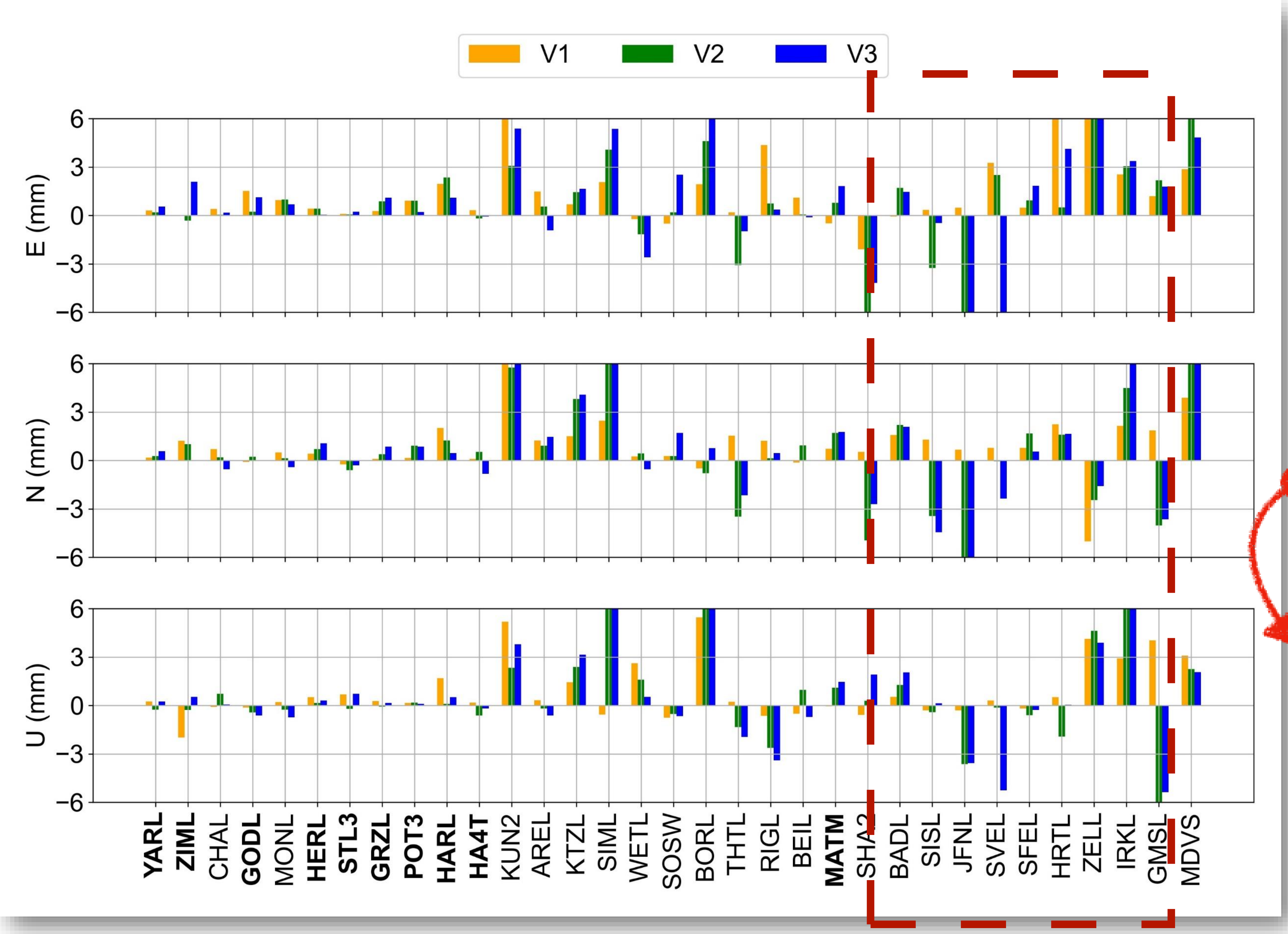


Figure: The repeatability of the station coordinates

Repeatability for different solution

Solution	All			Core			Non-core		
	E	N	U	E	N	U	E	N	U
V0	15.1	17.1	19.4	7.1	7.4	9.5	18.5	21.2	23.7
V1	14.2	15.7	18.8	6.4	7.0	9.4	17.5	19.5	22.9
V2	13.8	15.6	18.7	6.4	6.3	8.9	17.1	19.7	23.0
V3	13.6	15.2	18.6	6.0	6.2	8.9	16.9	19.1	22.9

- Overall, **V3 provides best performance**, improve the precision for all stations by **1mm**
- For the stations with insufficient observations, V3 induces a **deterioration** in precision

V0: All equal	V1: satellites	V2:stations	V3:satellite-station

SLR solution under different covariance matrices : sEOP

EOP w.r.t. the IERS-20-C04 series

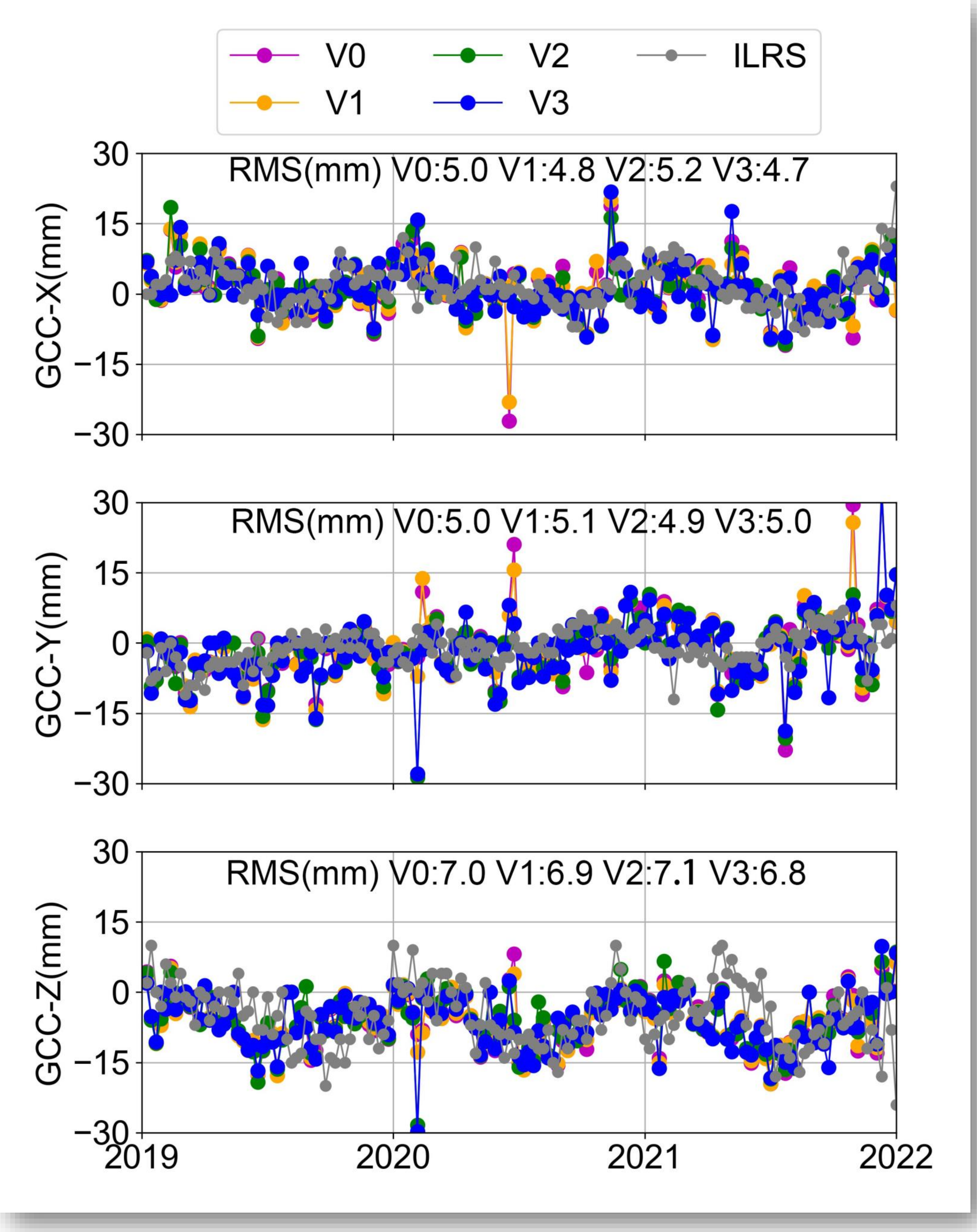
Solution	X pole (µas)			Y pole (µas)			LOD (µs/day)		
	MEAN	RMS	Outlier	MEAN	RMS	Outlier	MEAN	RMS	Outlier
V0	34.7	244.3	12	-17.5	225.8	14	4.7	47.9	17
V1	32.0	237.4	10	-20.0	223.1	12	5.1	45.2	11
V2	26.6	197.1	10	-15.4	186.3	7	3.2	42.5	11
V3	28.3	187.7	9	-12.8	175.8	7	1.9	37.4	12

- VCE provides more **concentrated** series with few outliers, improve the stability of the EOP investigation.

- **V3 gives the best performance**, decreasing the RMS of EOP by (23.1%, 22.1%, 21.9%) in Xpole, Ypole and ΔLOD components.

V0: All equal	V1: satellites	V2:stations	V3:satellite-station

SLR solution under different covariance matrices : GCC



Solution	X		Y		Z	
	Amp.(mm)	Phase(°)	Amp.(mm)	Phase(°)	Amp.(mm)	Phase(°)
ILRS	3.5	241	2.9	353	4.7	209
V0	2.8	242	2.9	273	4.6	247
V1	3.1	242	3.0	277	4.4	243
V2	2.9	239	2.5	265	4.3	245
V3	2.8	232	2.6	266	4.3	242

- VCE has a minor impact on the GCC estimation. RMS values of different solutions are at the same level with a difference within 1 mm.

V0: All equal	V1: satellites	V2:stations	V3:satellite-station

The effect of the observation division: Conclusion

Result

The precision of station coordinate

The precision of EOP

- V3 (division by **satellite-station pair**) is the most realistic model for VCE, since it counts for the **discrepancy** in quality of the observations from different satellites and stations

V0: All equal	V1: satellites	V2:stations	V3:satellite-station

The effect of the observation division: Conclusion

Result

The precision of station coordinate

The precision of EOP

→

- V3 (division by **satellite-station pair**) is the most realistic model for VCE, since it counts for the **discrepancy** in quality of the observations from different satellites and stations

Problem

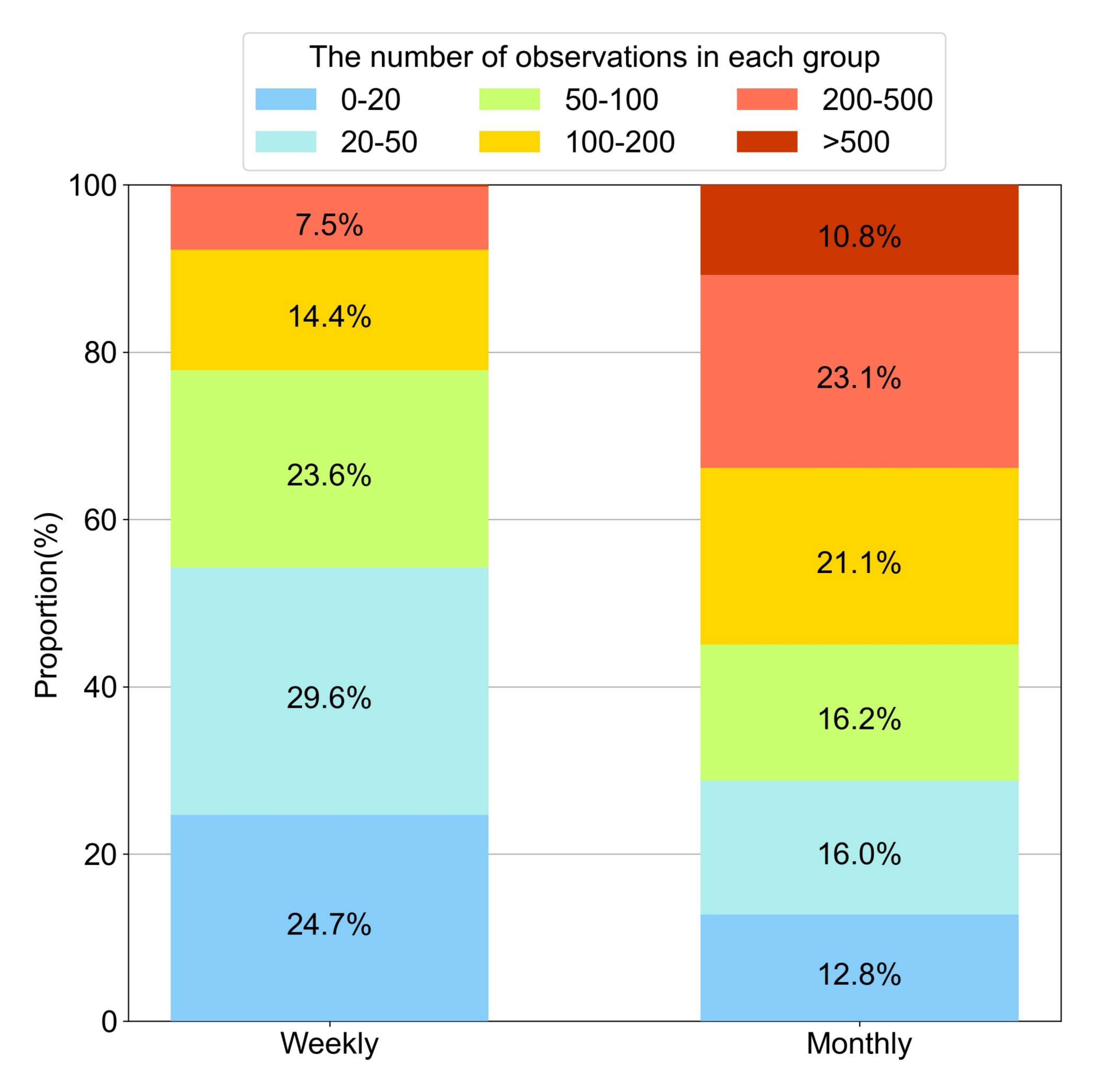
V3 scheme → **insufficiency in some groups** → **precision deterioration**

- The groups with **insufficient** observations limits the performance of the VCE under V3

V0: All equal	V1: satellites	V2:stations	V3:satellite-station

SLR solution under monthly weights : Strategy

How to eliminate the undesirable effect caused by **insufficiency**?

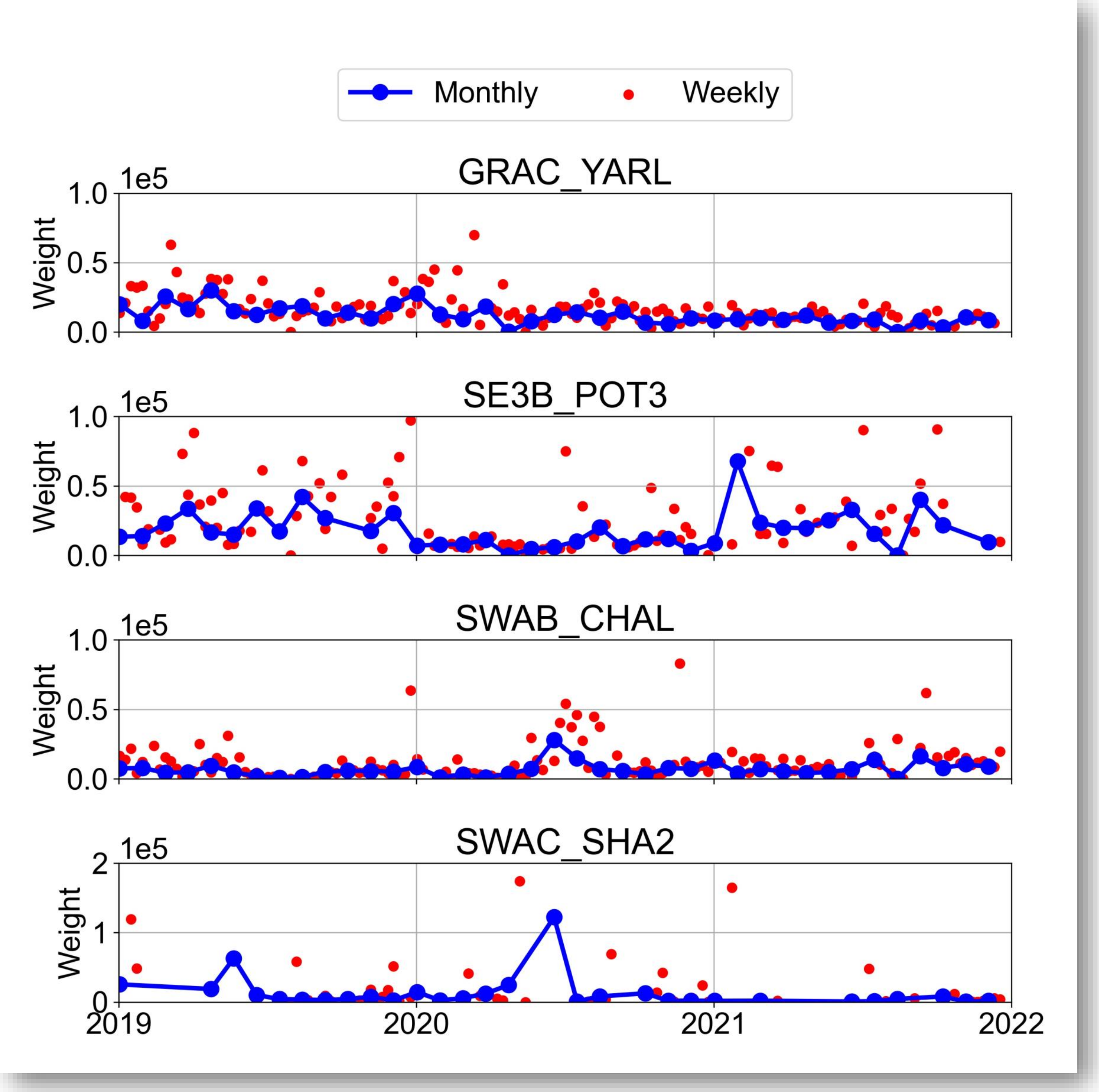


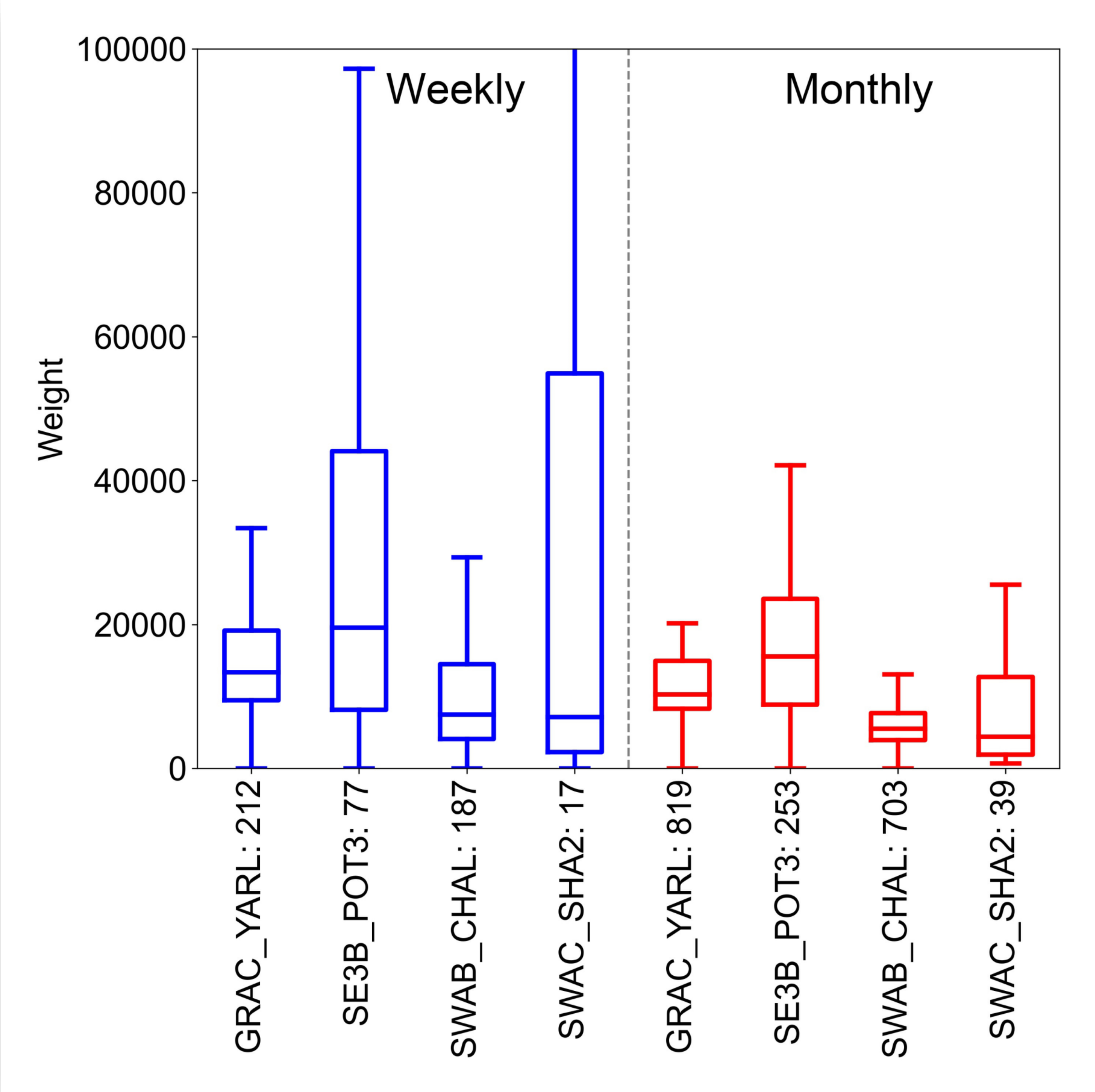
We introduce **monthly solution** for the VCE with the observation division by satellite-stations pair, and use the derived weight in the **7-day solution**.

Scheme	Description
V3	The observation weighted by VCE are divided based on station and satellited
V3_M	The observation weighted by VCE are divided based on station and satellited in monthly solution

- Monthly solution evidently **increase the quantity** of the observations in one group

SLR solution under monthly weights : Derived weight





- The weight series derived by 7-day and monthly solution show **consistency**.
- The monthly solution **decrease the IQR** of the weight in the group insufficient with observations

V3: weight derived by 7-day solution	V3_M: weight derived by monthly solution

SLR solution under monthly weights : Station coordinates

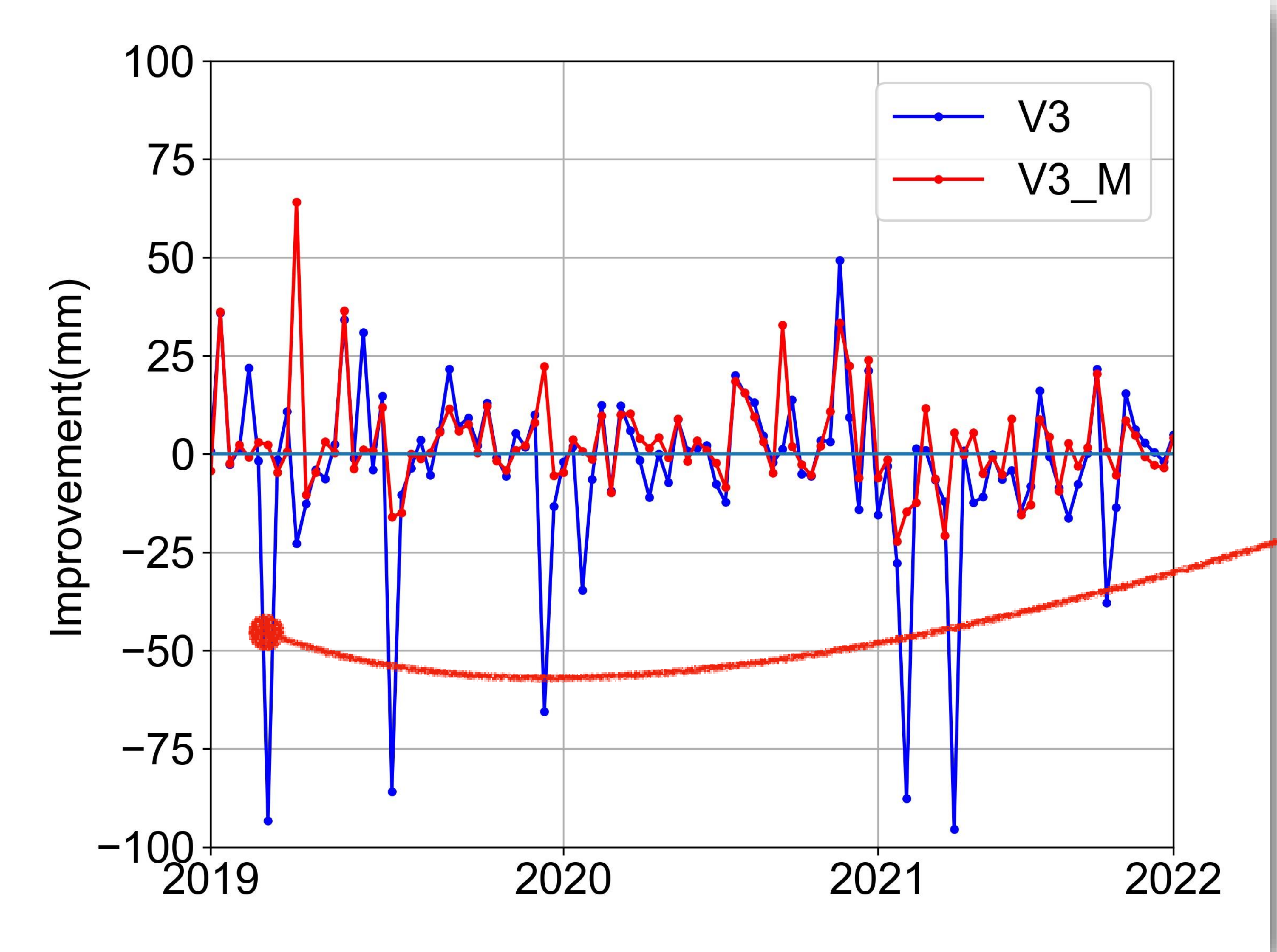


Figure: The improvement series to the result of equally weight for a station with insufficient observations under different scheme

- The introduction of the monthly weight **corrects** most deteriorations caused by the insufficiency
- The average improvement by VCE for that station increase from **-3.4 mm to 2.9** mm by V3_M

V3: weight derived by 7-day solution	V3_M: weight derived by monthly solution

SLR solution under monthly weights: Station coordinates

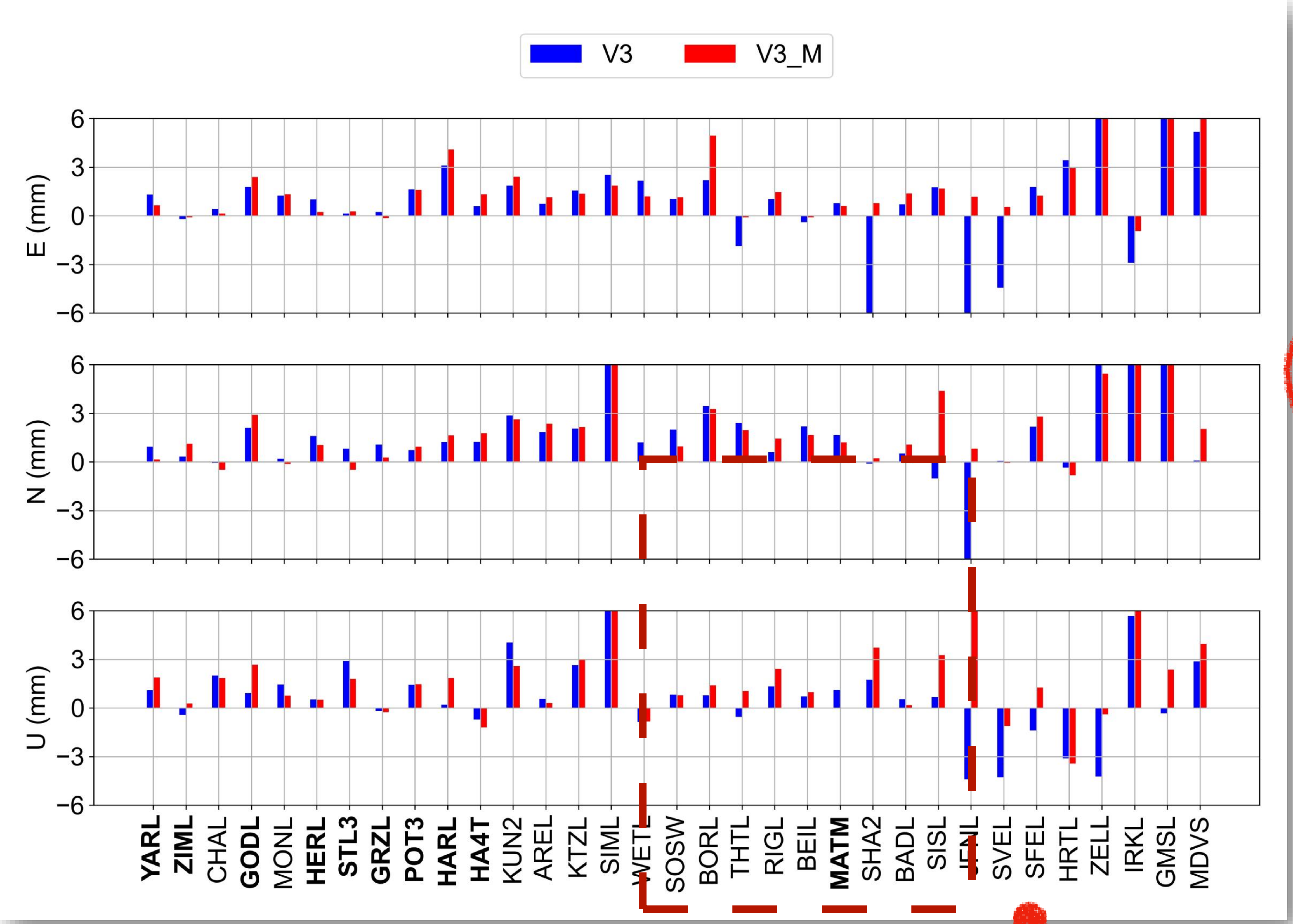


Figure: The repeatability of the station coordinates

Solution	All			Core			Non-core		
	E	N	U	E	N	U	E	N	U
V3	13.6	15.2	18.6	6.0	6.2	8.9	16.9	19.1	22.9
V3_M	12.7	14.6	17.6	6.0	6.3	8.7	15.8	18.3	21.6

- The contribution of importing the monthly weight is significant for the non-core stations.
- The RMS of stations suffering from the insufficient observations in a group **improve significantly**

V3: weight derived by 7-day solution	V3_M: weight derived by monthly solution

Conclusion

- VCE **optimizes** the combination of multi-LEOs and **improves** the SLR-derived geodetic parameters.

- The structure based on **satellite-station pair** is the most reasonable way to build the covariance matrix .

- It is possible to **eliminate** the VCE deterioration due to the insufficient observations by importing the weight from a **monthly solution**.

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GREAT

GNSS+ REsearch, Application and Teaching

https://github.com/GREAT-WHU

● Software function

✓Generate precise products of orbit, clock, UPD, ionosphere, and tropospher

✓Joint solution of GNSS/SLR /VLBI

✓LEO enhanced GNSS

✓Real-time precise positioning, such as PPP-RTK/PPP/NRTK

✓Multi-source fusion navigation with GNSS/INS/VISION/LiDAR

✓……

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Thank you

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