

Diego Blas Temiño (on behalf of A. Jenkins, M. Herrero, X. Xue, A. Bourguin, A. Hess, J. Foster)  
[dblasis@ifae.es](mailto:dblasis@ifae.es)



Detecting gravitational waves  
with SLR

# Published works

<https://arxiv.org/abs/2107.04601>  
<https://arxiv.org/abs/2107.04063>

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103

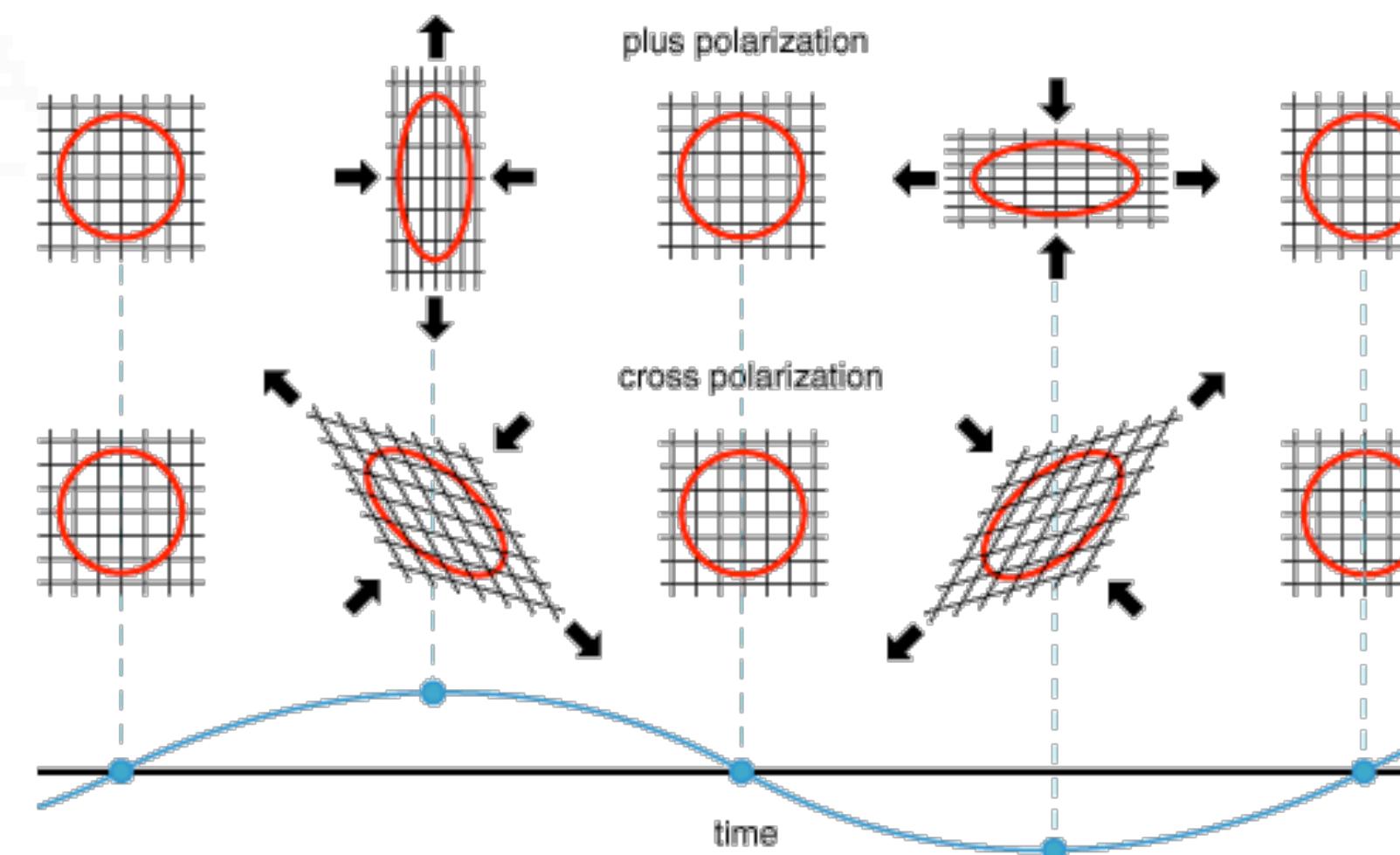
Blas&Jenkins Phys.Rev.D 105 (2022) 6, 064021

+ *new papers to appear*

# GWs (essentials)

Perturbations of space-time  
travelling as waves of frequency  $f$

Characterised by 2 polarizations  $h_{+,\times}$  (dimensionless)



$$c = 1 \\ h_{+,\times} \approx h_0 \cos(2\pi f(t - z) + \phi)$$

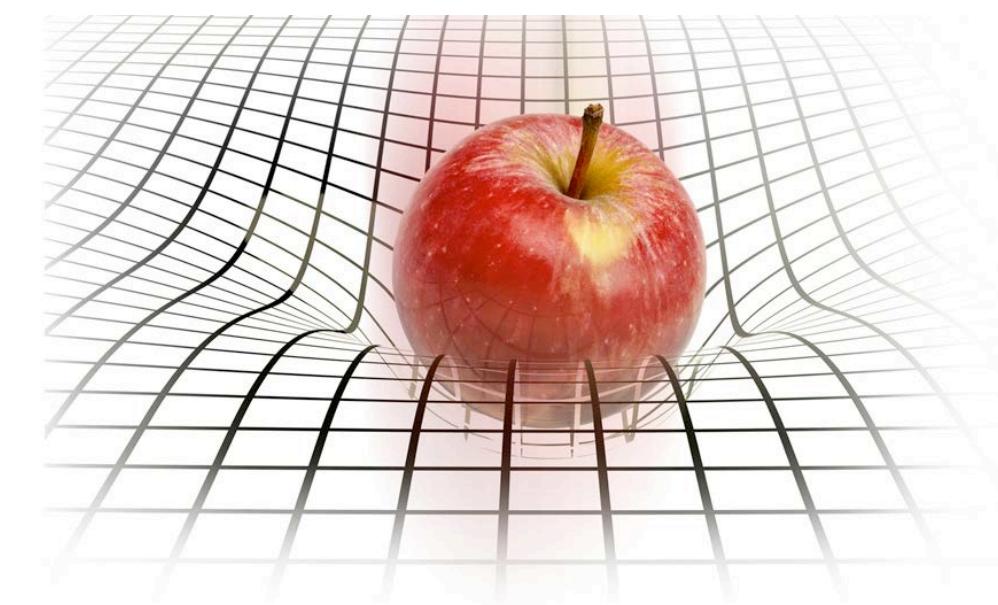
GWs carry energy. They have **energy density**

$$\rho_{\text{gw}} = \frac{1}{16\pi G} \langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle \rightarrow \Omega_{\text{gw}}(f) \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{gw}}}{d(\ln f)}$$

$$\rho_c = 1.2 \times 10^{11} M_\odot \text{Mpc}^{-3} \sim \text{keV/cm}^3$$

$$h \approx 0.67$$

# Taxonomy of GWs



$h(t)$   
phase  
modelled

transient

persistent

continuous waves

BH binaries

pulsars

bursts

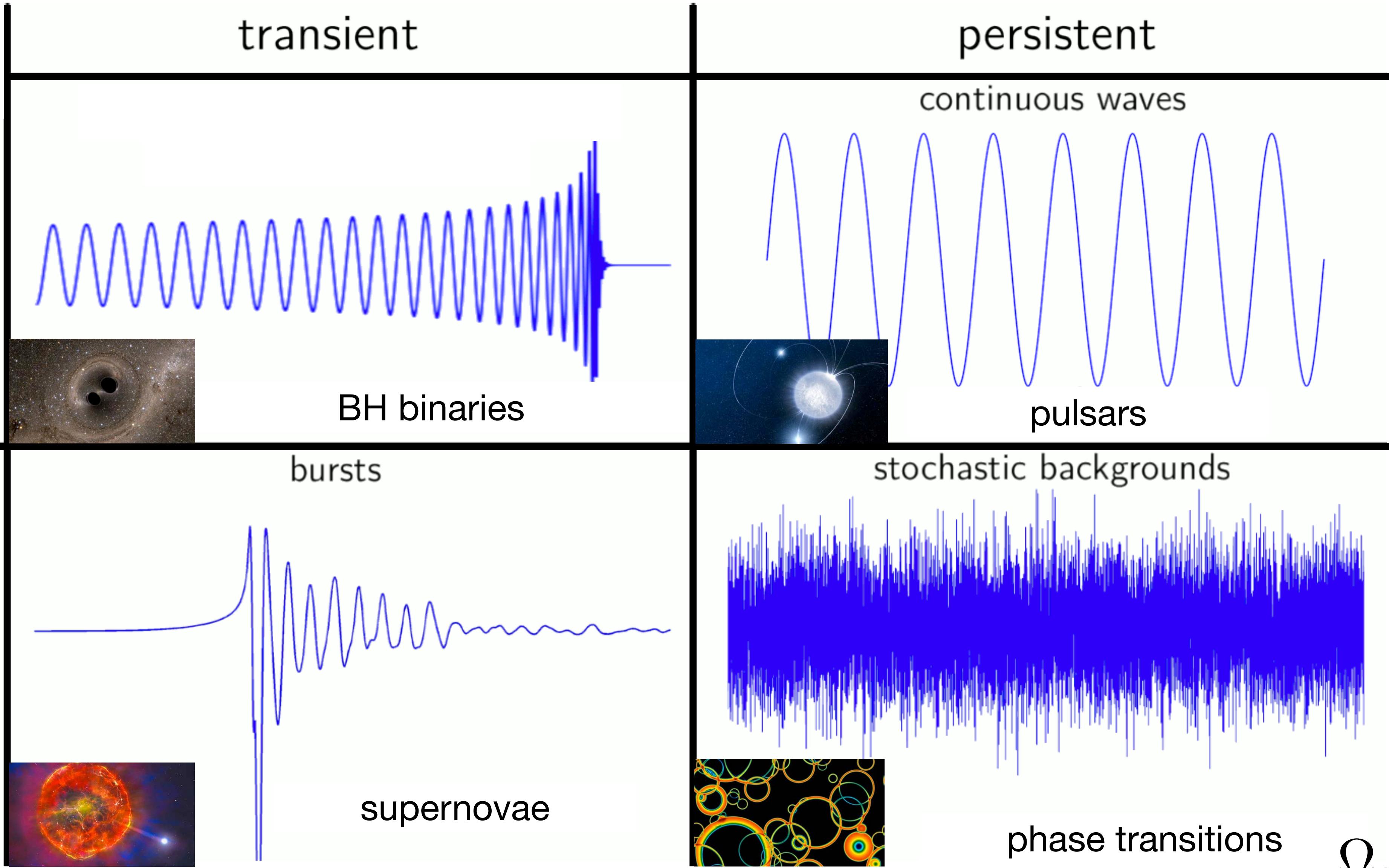
stochastic backgrounds

phase  
unmodelled

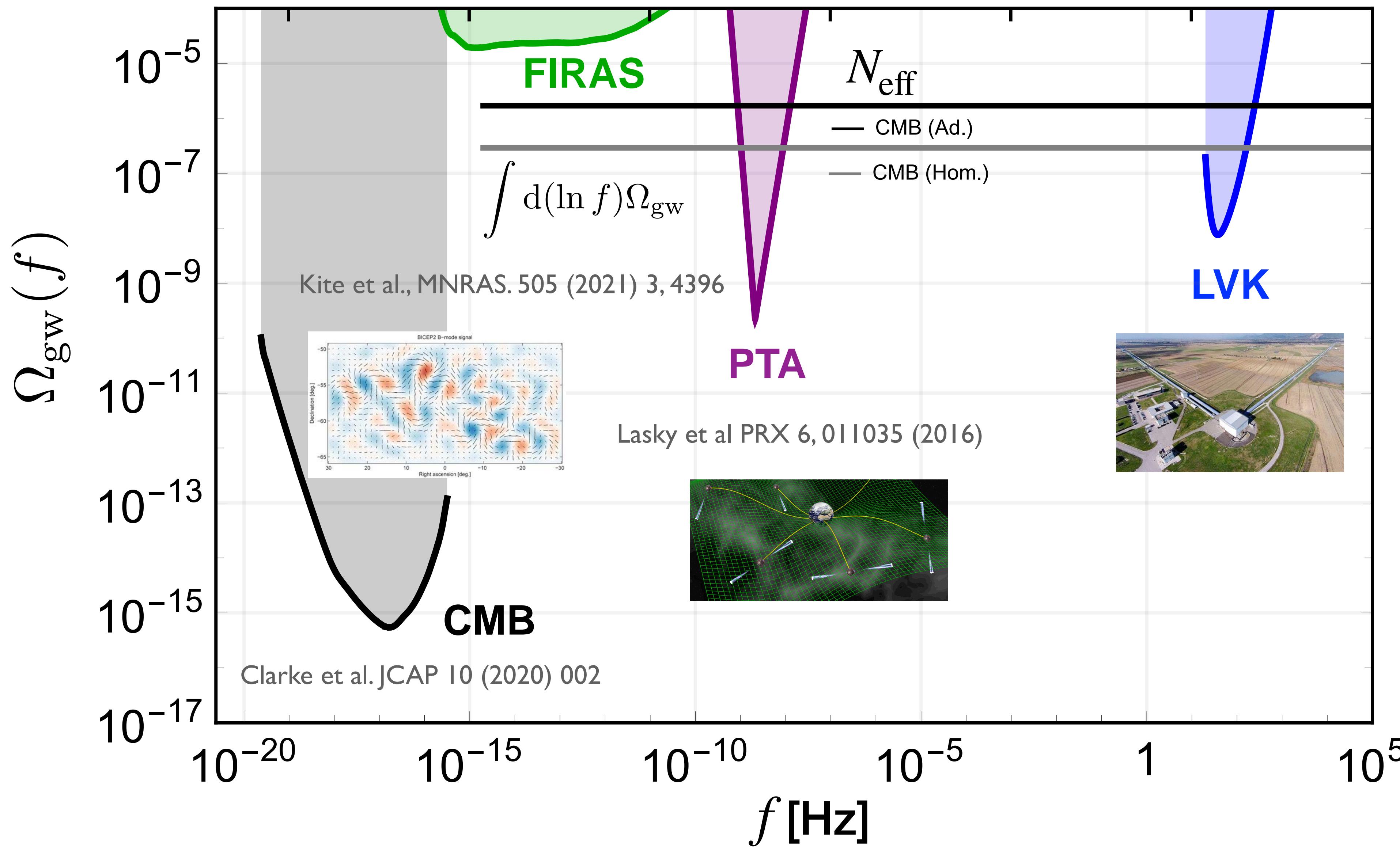
supernovae

phase transitions

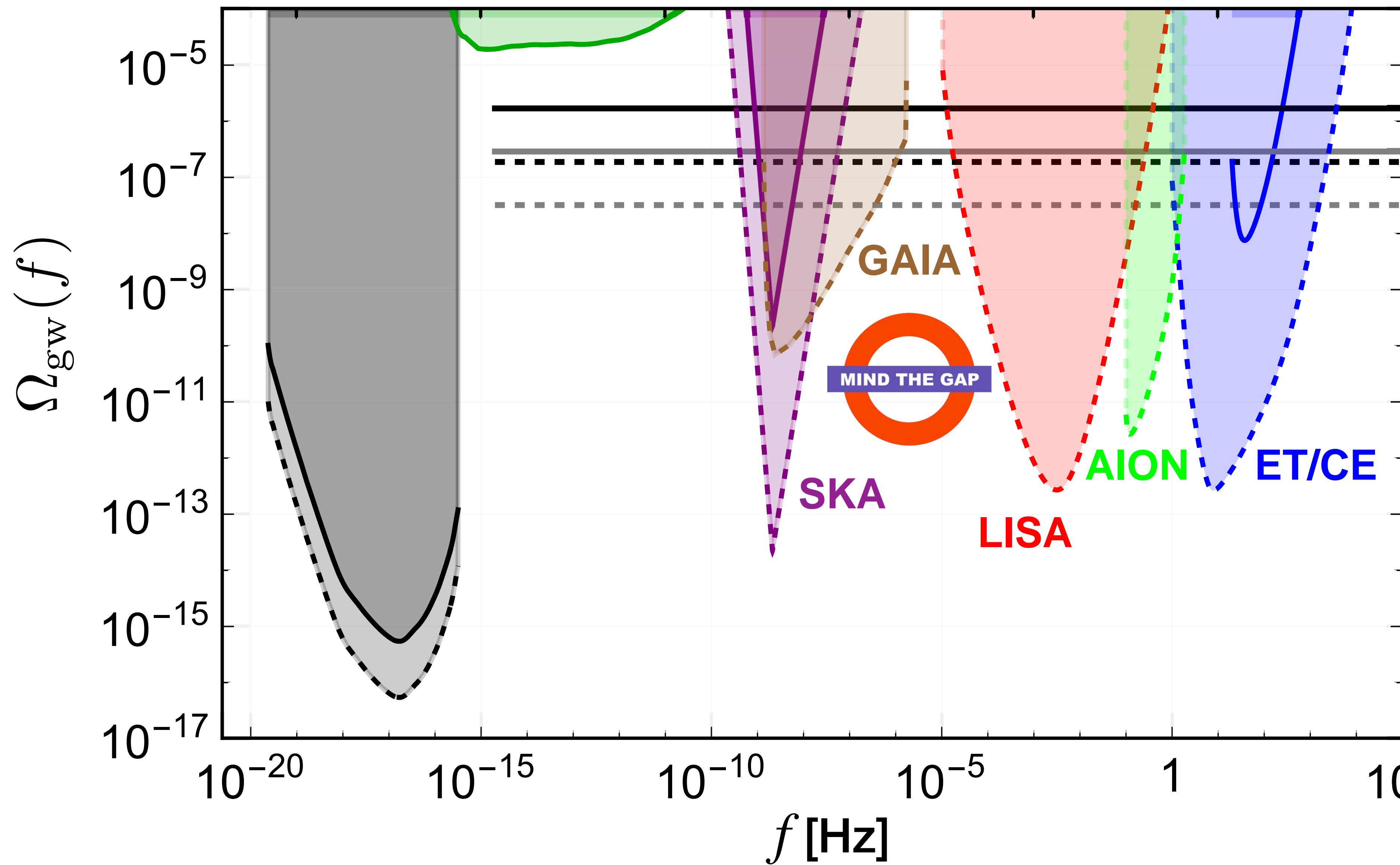
$\Omega_{\text{gw}}(f)$



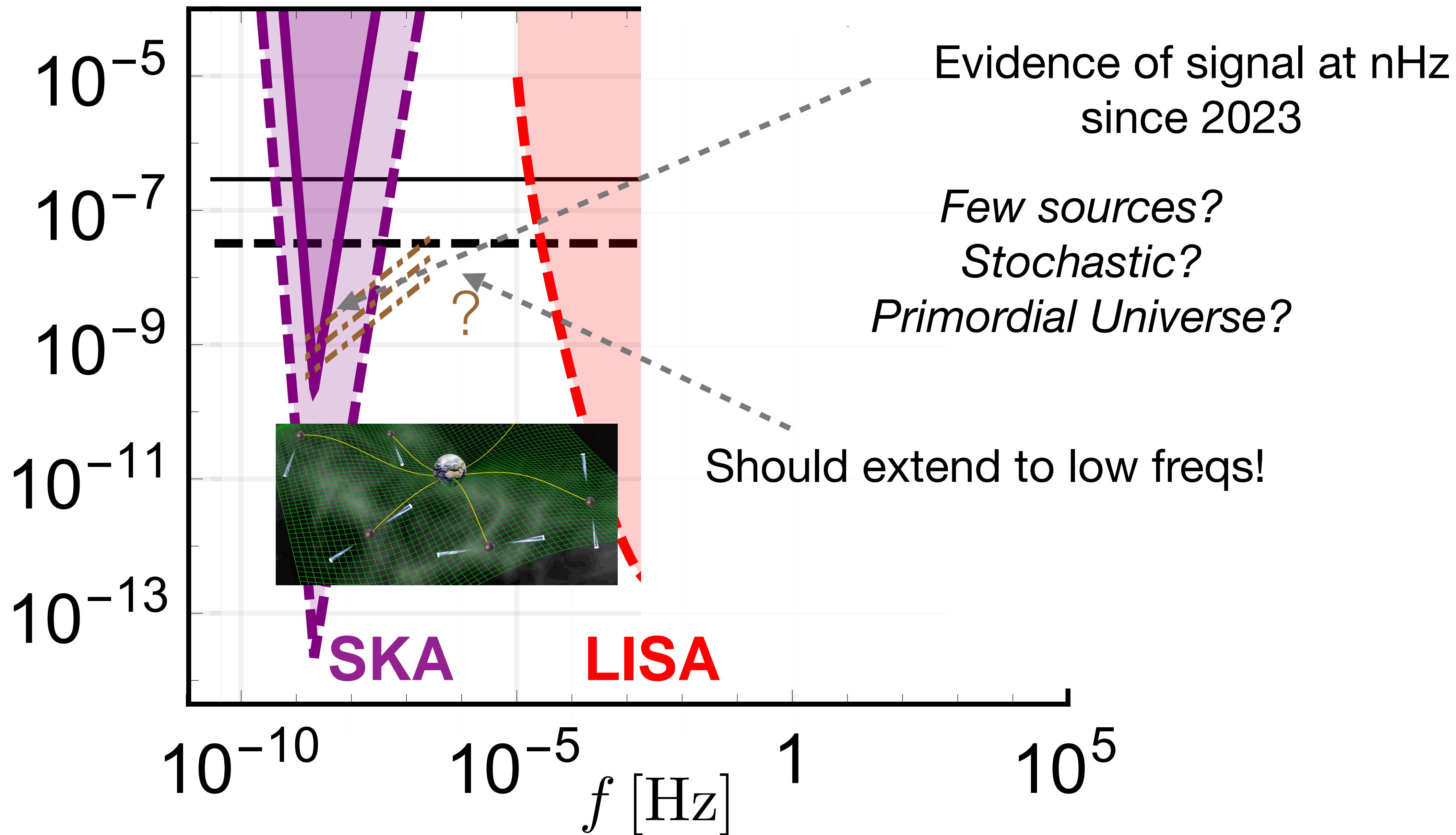
# GWs landscape today



# GWs landscape ca. 2040



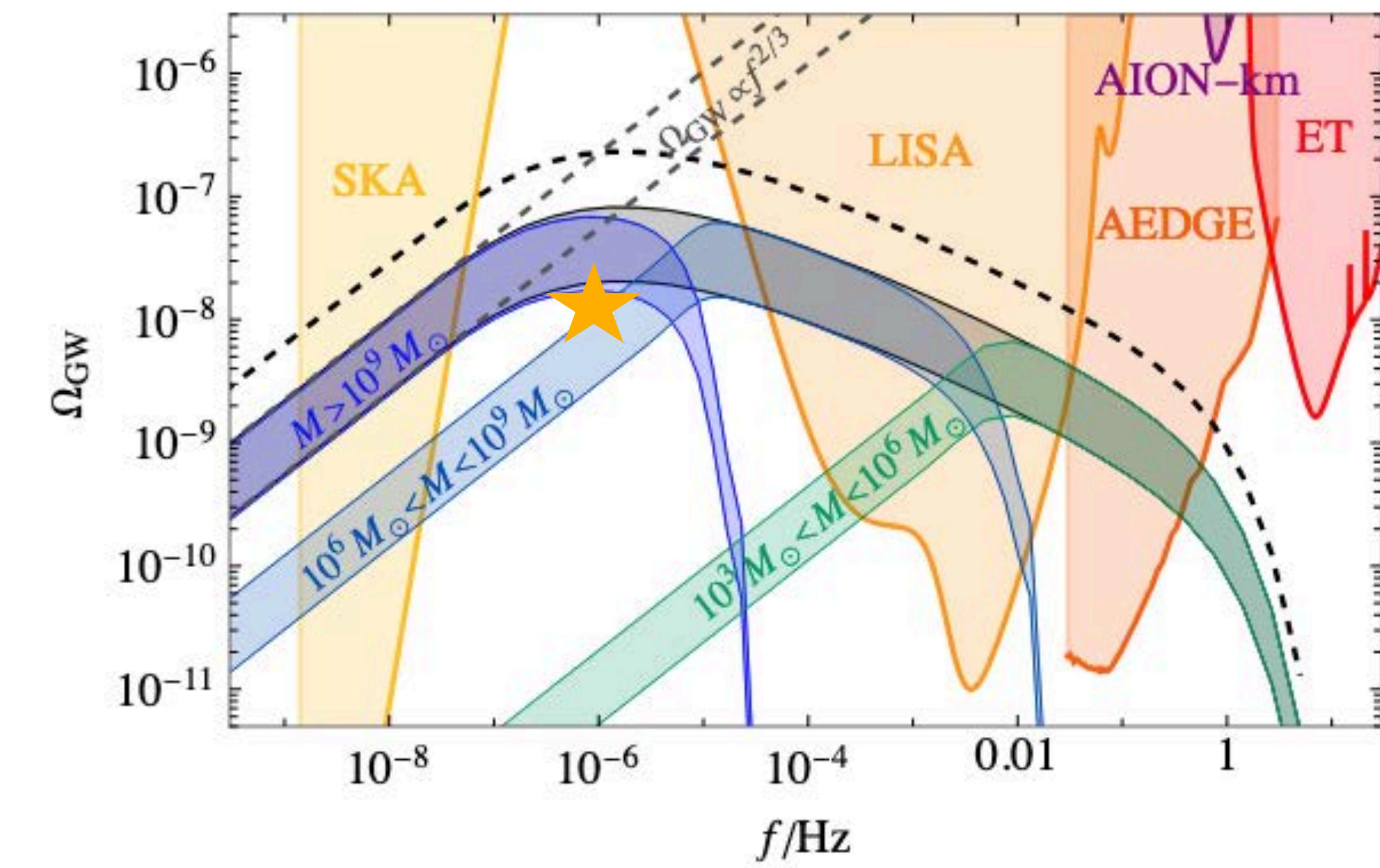
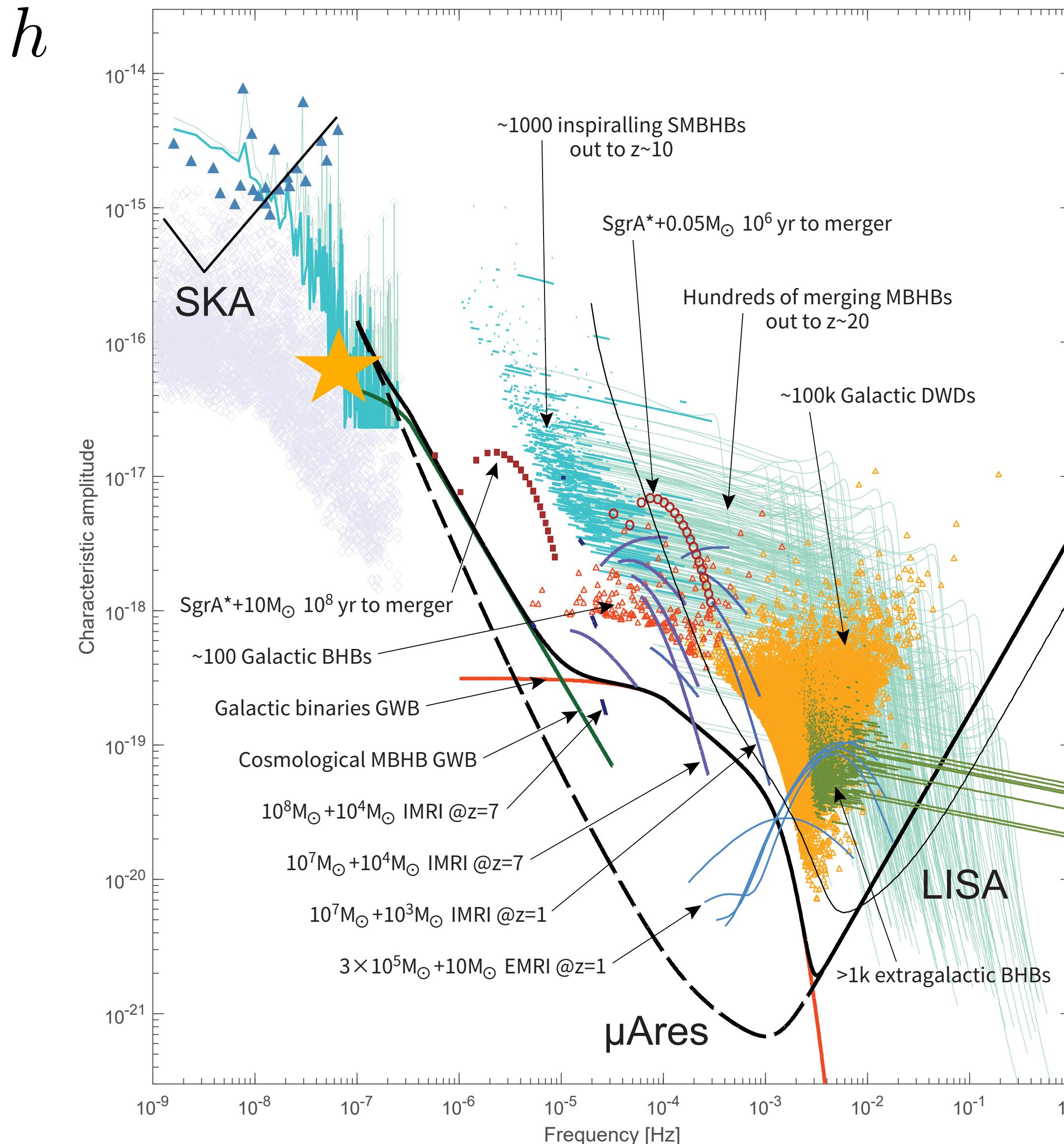
# Possible backgrounds at $\mu\text{Hz}$ : a rich band



# Review of sources

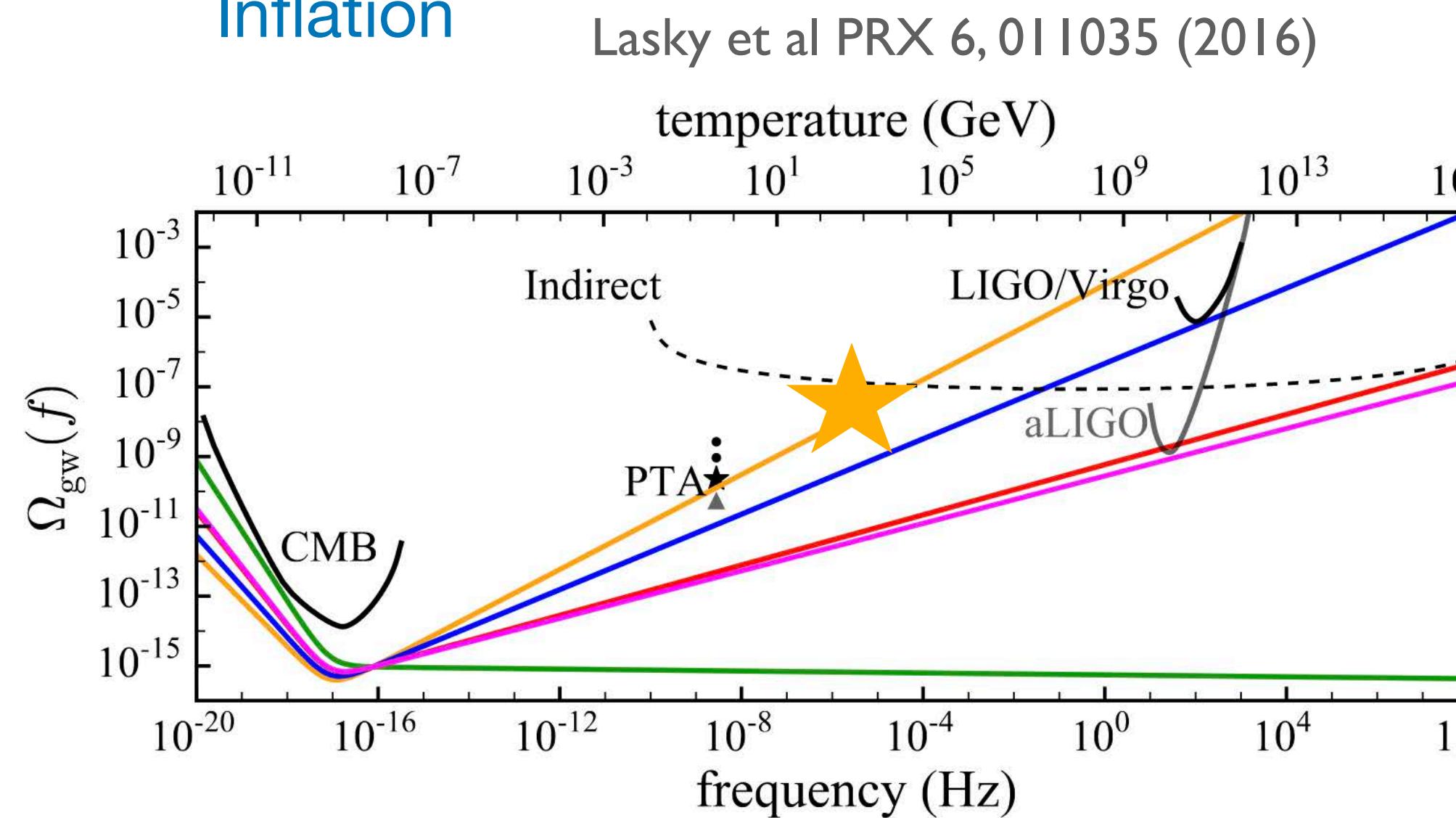
arXiv:1908.11391v1 [astro-ph.IM] 29 Aug 2019

## The $\mu$ Ares detection landscape

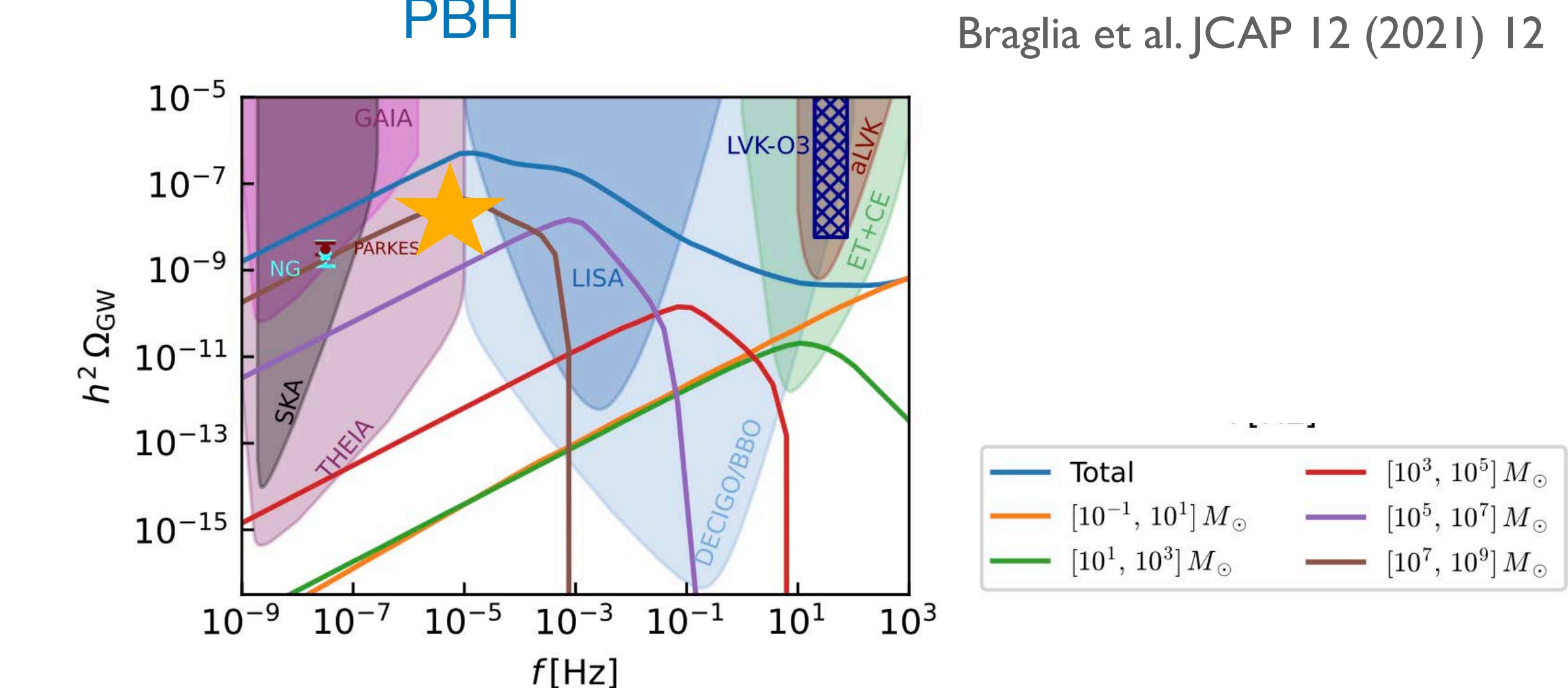


# Backgrounds from fundamental physics

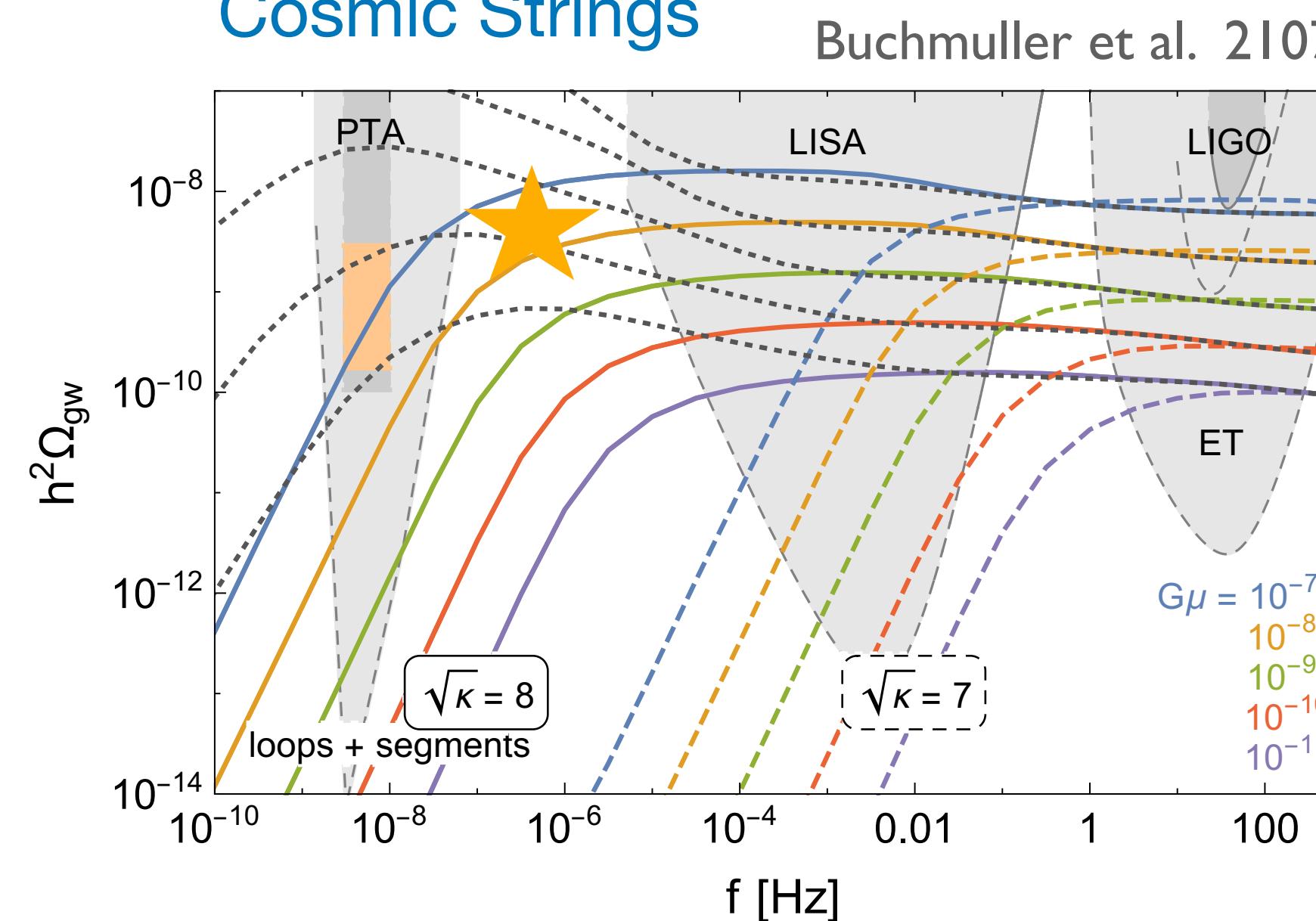
## Inflation



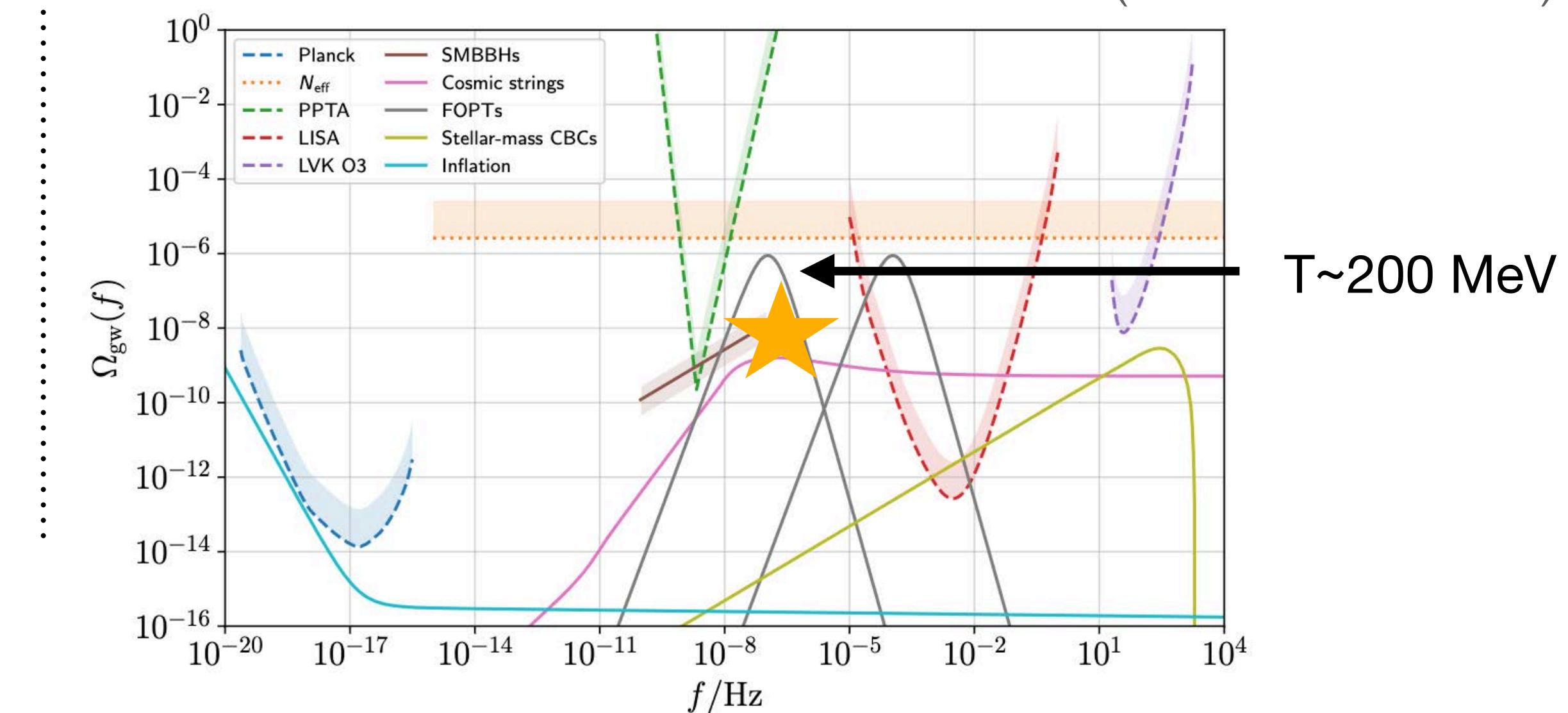
## PBH



## Cosmic Strings



## FOPT



# Absorption of GWs by binaries

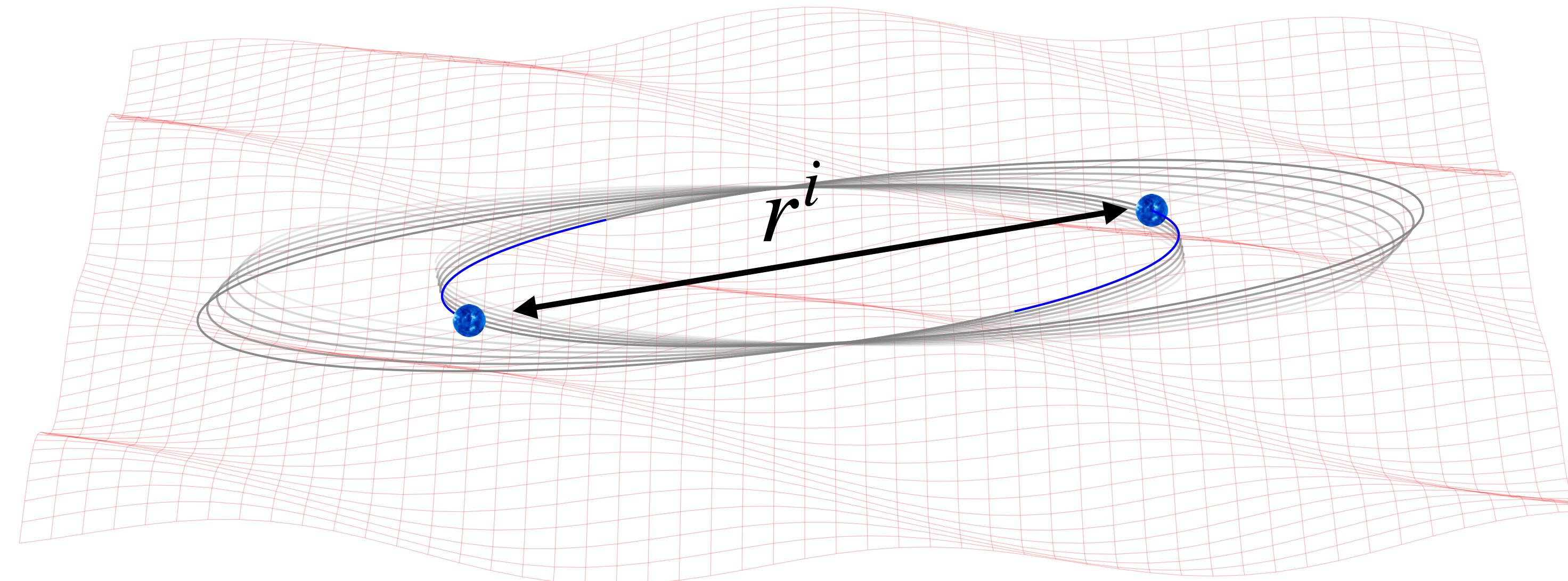
$f \sim \mu\text{Hz}$   
few days

## Intuitive idea (from '60s)

Influence of a GW on a binary system (e.g. non-relativistic)

$$\ddot{r}^i + \frac{GM}{r^3}r^i = \delta^{ik}\frac{1}{2}\ddot{h}_{kj}r^j$$

Newtonian potential     ...  
GW

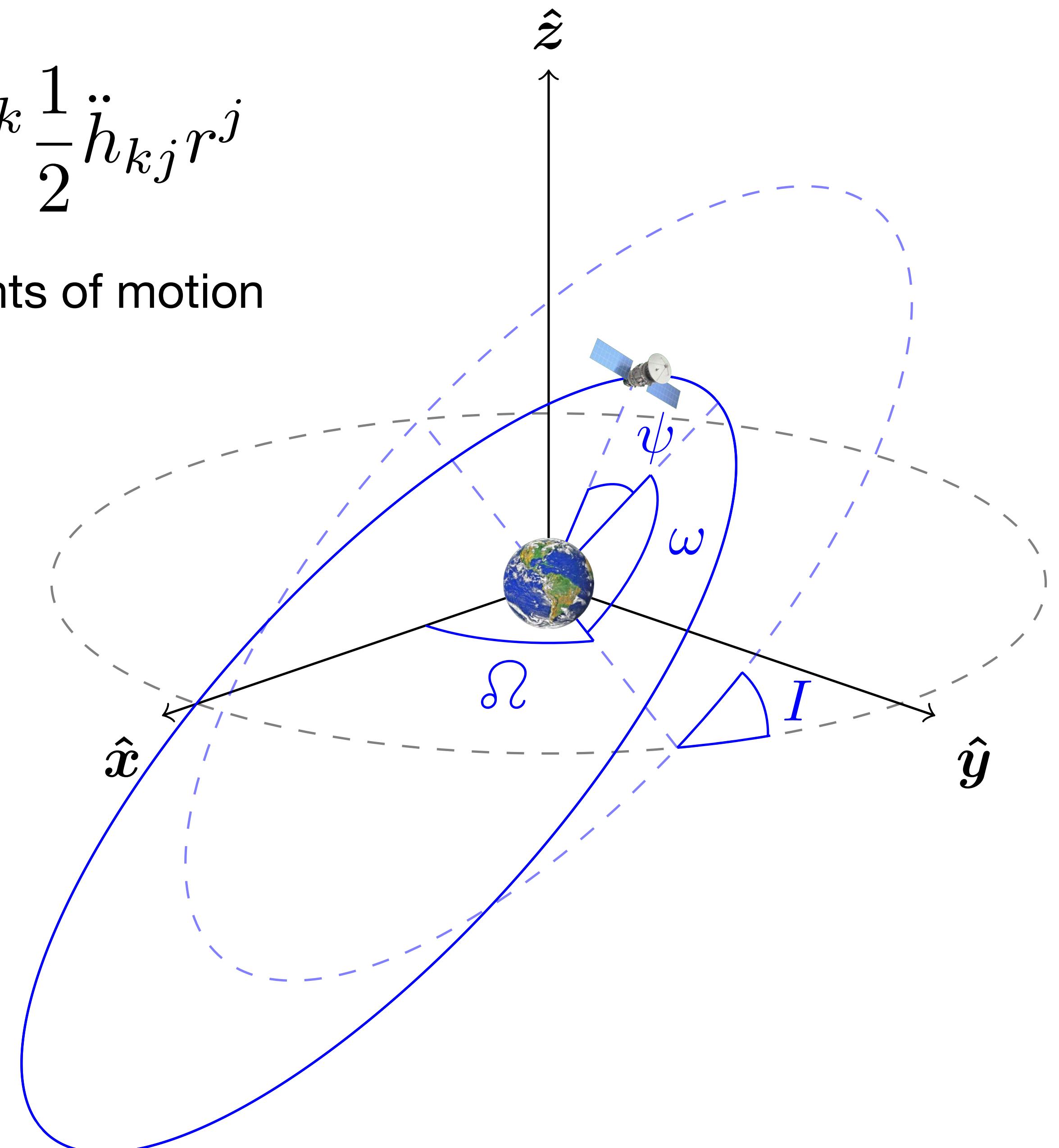


# Absorption of GWs by binaries

$$\ddot{r}^i + \frac{GM}{r^3} r^i = \delta^{ik} \frac{1}{2} \ddot{h}_{kj} r^j$$

Better characterised for its 6 Newtonian constants of motion

- **period  $P$ , eccentricity  $e$ :**  
*size and shape of orbit*
- **inclination  $I$ , ascending node  $\Omega$ :**  
*orientation in space*
- **pericentre  $\omega$ ,**  
**mean anomaly at epoch  $\varepsilon$ :**  
*radial and angular phases*



# Absorption of GWs by binaries

$$\ddot{\mathbf{r}} + \frac{GM}{r^2} \hat{\mathbf{r}} = \delta \ddot{\mathbf{r}}.$$

■ for generic perturbation:

$$\delta \ddot{\mathbf{r}} = r(\mathcal{F}_r \hat{\mathbf{r}} + \mathcal{F}_\theta \hat{\boldsymbol{\theta}} + \mathcal{F}_\ell \hat{\boldsymbol{\ell}}),$$



$$\begin{aligned}\dot{P} &= \frac{3P^2\gamma}{2\pi} \left[ \frac{e \sin \psi \mathcal{F}_r}{1 + e \cos \psi} + \mathcal{F}_\theta \right], \\ \dot{e} &= \frac{\dot{P}\gamma^2}{3Pe} - \frac{P\gamma^5 \mathcal{F}_\theta}{2\pi e (1 + e \cos \psi)^2}, \\ \dot{I} &= \frac{P\gamma^3 \cos \theta \mathcal{F}_\ell}{2\pi (1 + e \cos \psi)^2}, \\ \dot{\Omega} &= \frac{\tan \theta}{\sin I} \dot{I}, \\ \dot{\omega} &= \frac{P\gamma^3}{2\pi e} \left[ \frac{(2 + e \cos \psi) \sin \psi \mathcal{F}_\theta}{(1 + e \cos \psi)^2} - \frac{\cos \psi \mathcal{F}_r}{1 + e \cos \psi} \right] - \cos I \dot{\Omega}, \\ \dot{\varepsilon} &= -\frac{P\gamma^4 \mathcal{F}_r}{\pi (1 + e \cos \psi)^2} - \gamma (\cos I \dot{\Omega} + \dot{\omega}),\end{aligned}$$

Notice resonance effects

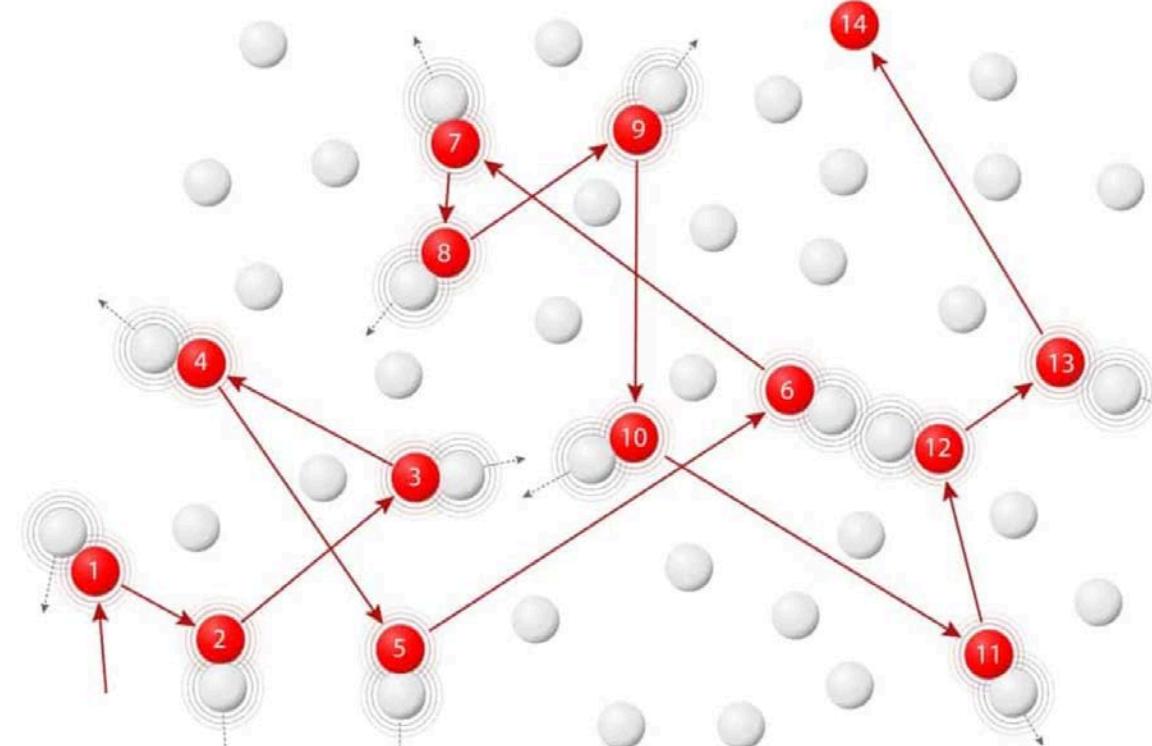
$$\langle \sin(\omega t) \sin(\omega_{gw} t + \phi_0) \rangle \sim \frac{1}{2} \delta(\omega - \omega_{gw})$$

# Absorption of GWs by binaries

$$\ddot{\mathbf{r}} + \frac{GM}{r^2} \hat{\mathbf{r}} = \delta \ddot{\mathbf{r}}.$$

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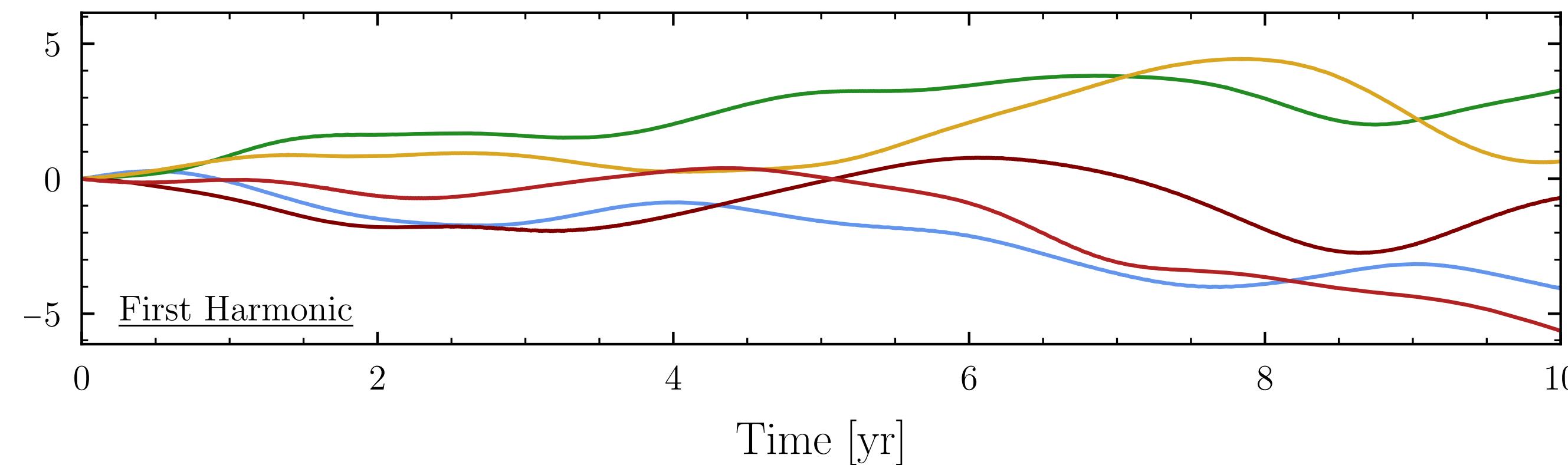
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Notice resonance effects

$$\langle \sin(\omega t) \sin(\omega_{gw} t + \phi_0) \rangle \sim \frac{1}{2} \delta(\omega - \omega_{gw})$$

# Confirming with simulations

Semi-latus Rectum Perturbation [cm]

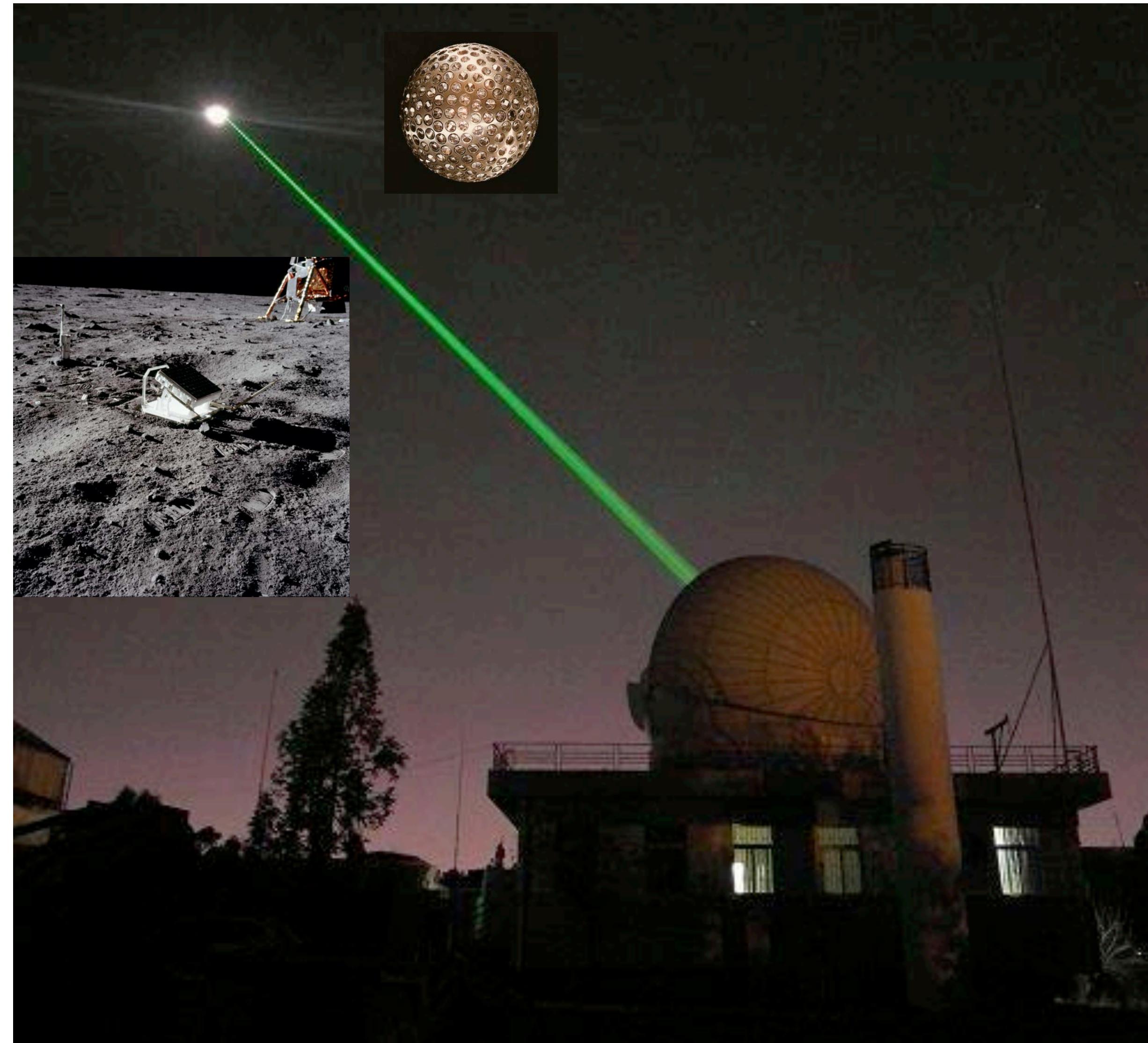


Credit: J. Foster

(work in progress: Blas, Bourguin, Foster, Hees, Herrero, Jenkins)

# Summary results for SLR and LLR

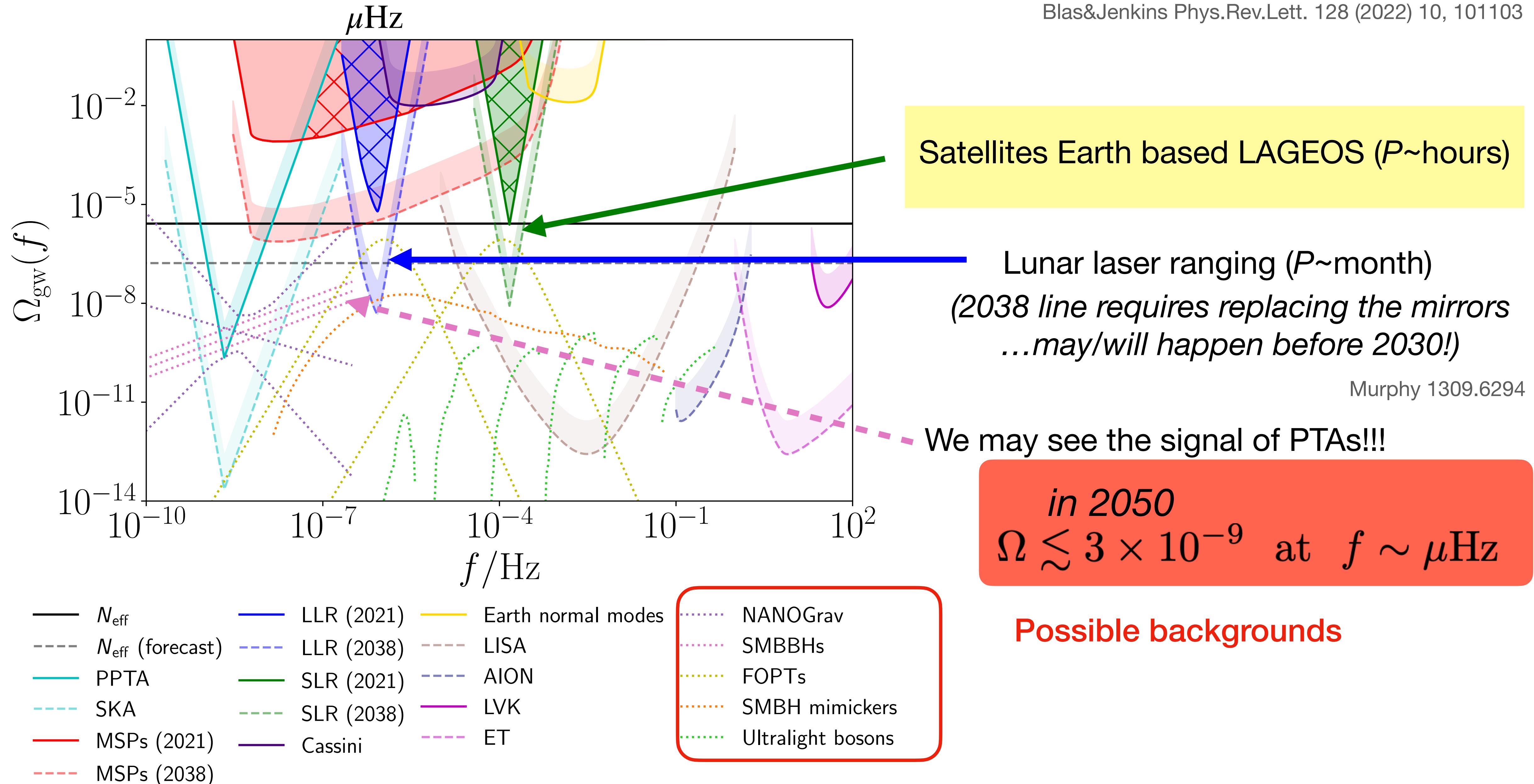
$f \sim \mu\text{Hz}$   
few days



# Our estimates from 2021 for 2038

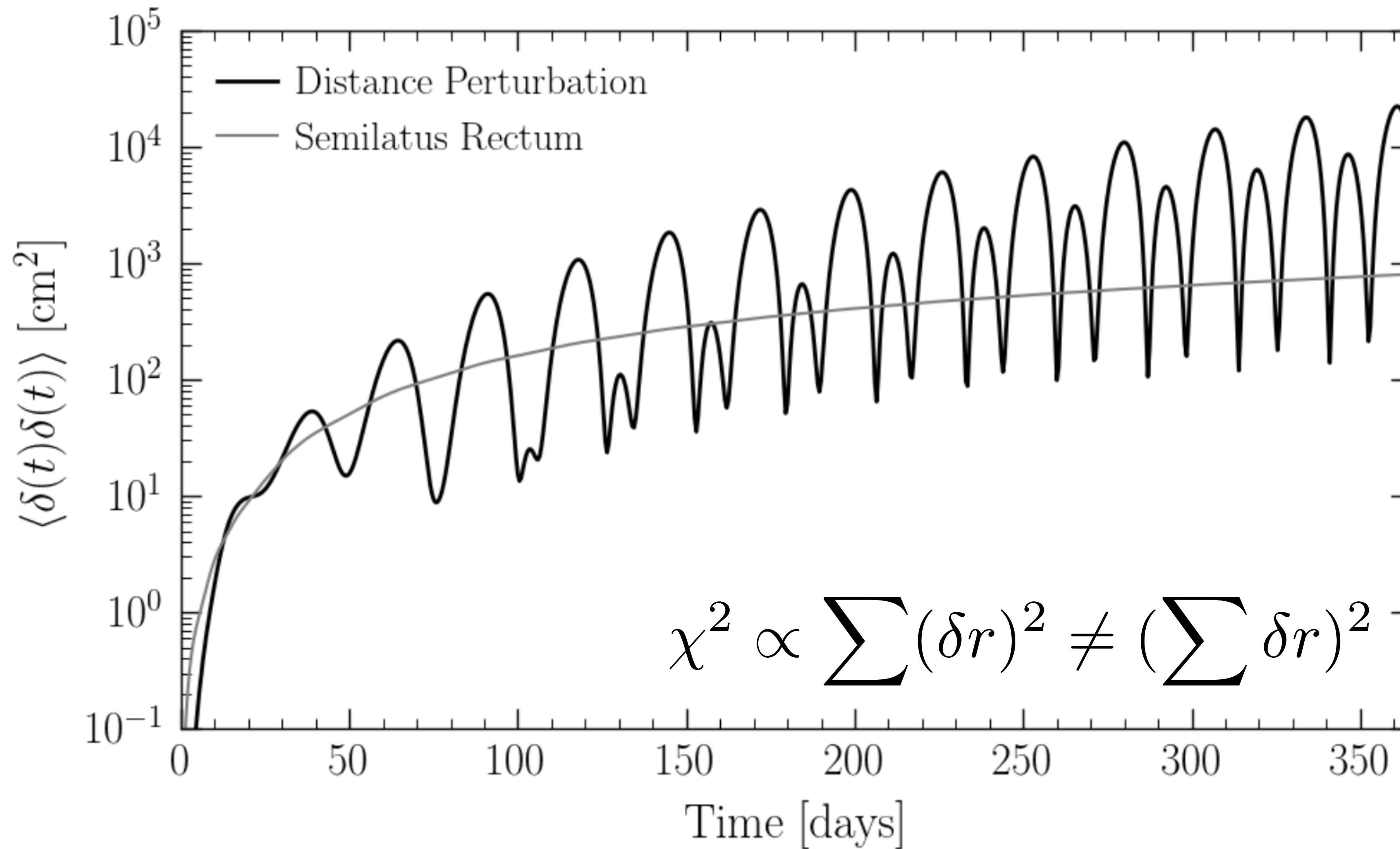
# Our estimates from 2021 for 2038

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



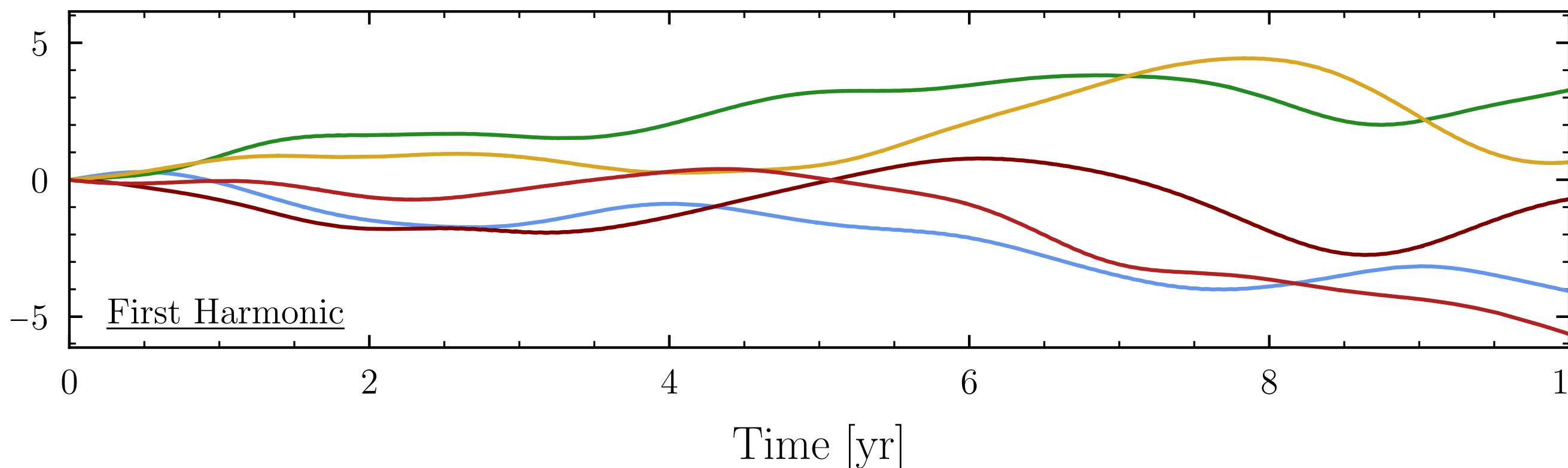
# New results (to appear)

# COMPARISON WITH PRIOR RESULTS (not using all the orbital info)



$$r = \frac{p}{1 + e \cos(f)}$$

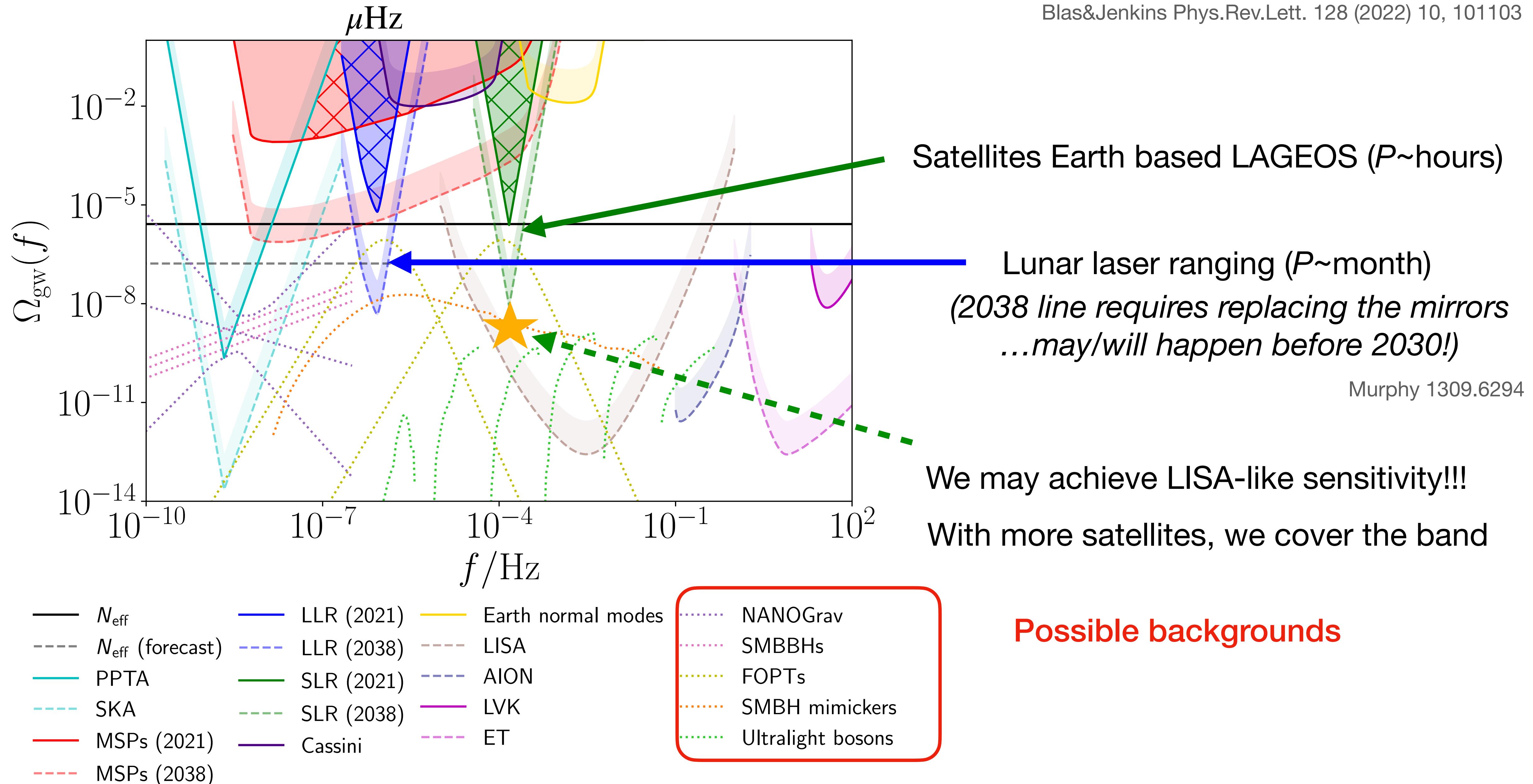
# New developments (using all the orbital info)



- **all the information of the orbit** (not only secular effects)
  - **O(1000) better sensitivity**
  - We need large **P, e** and **good orbital data**
- Better SLR at the Moon??

# Our estimates from 2021 for 2038

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# Example for SLR in Moon orbits

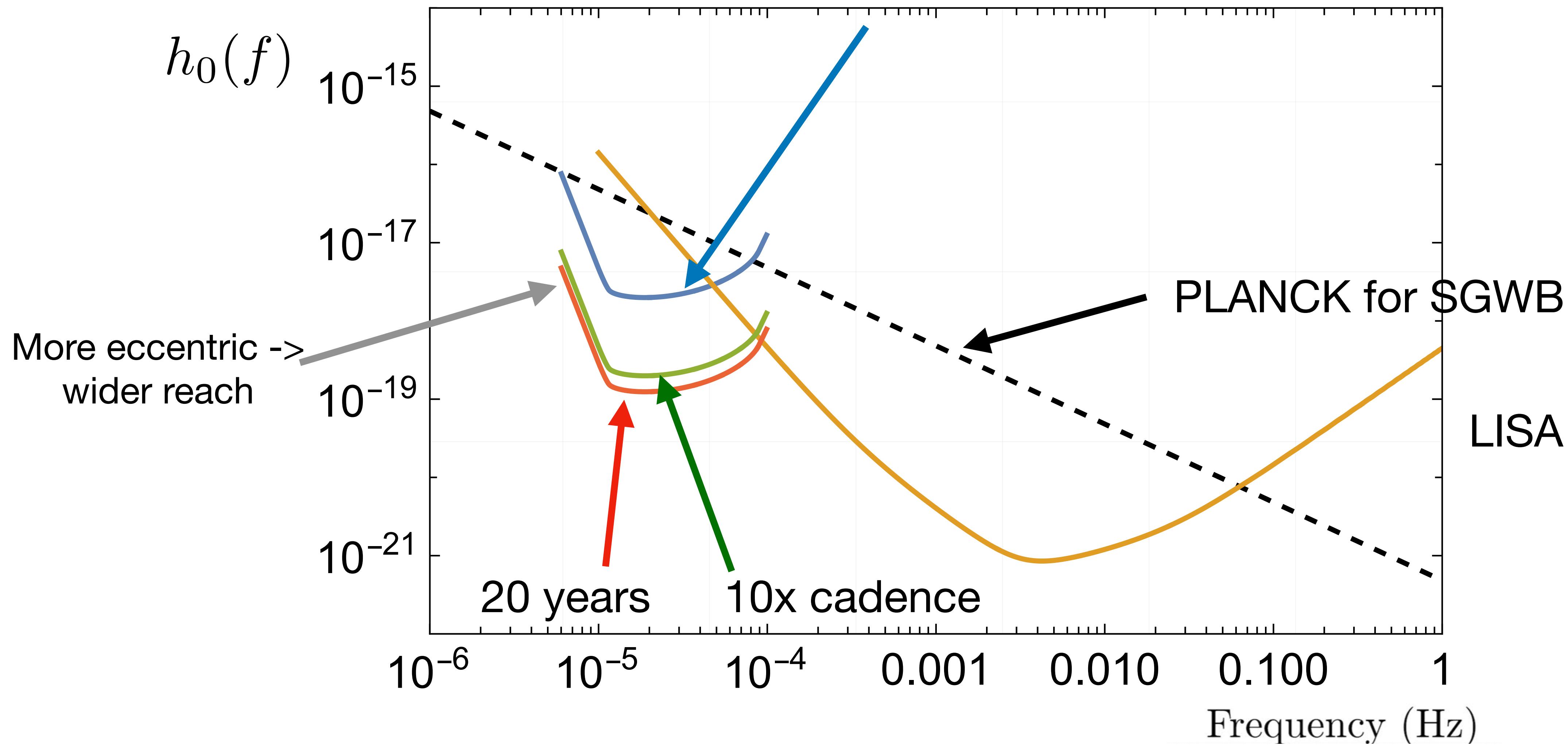
10 years, 1000 points/ year, 1cm/point

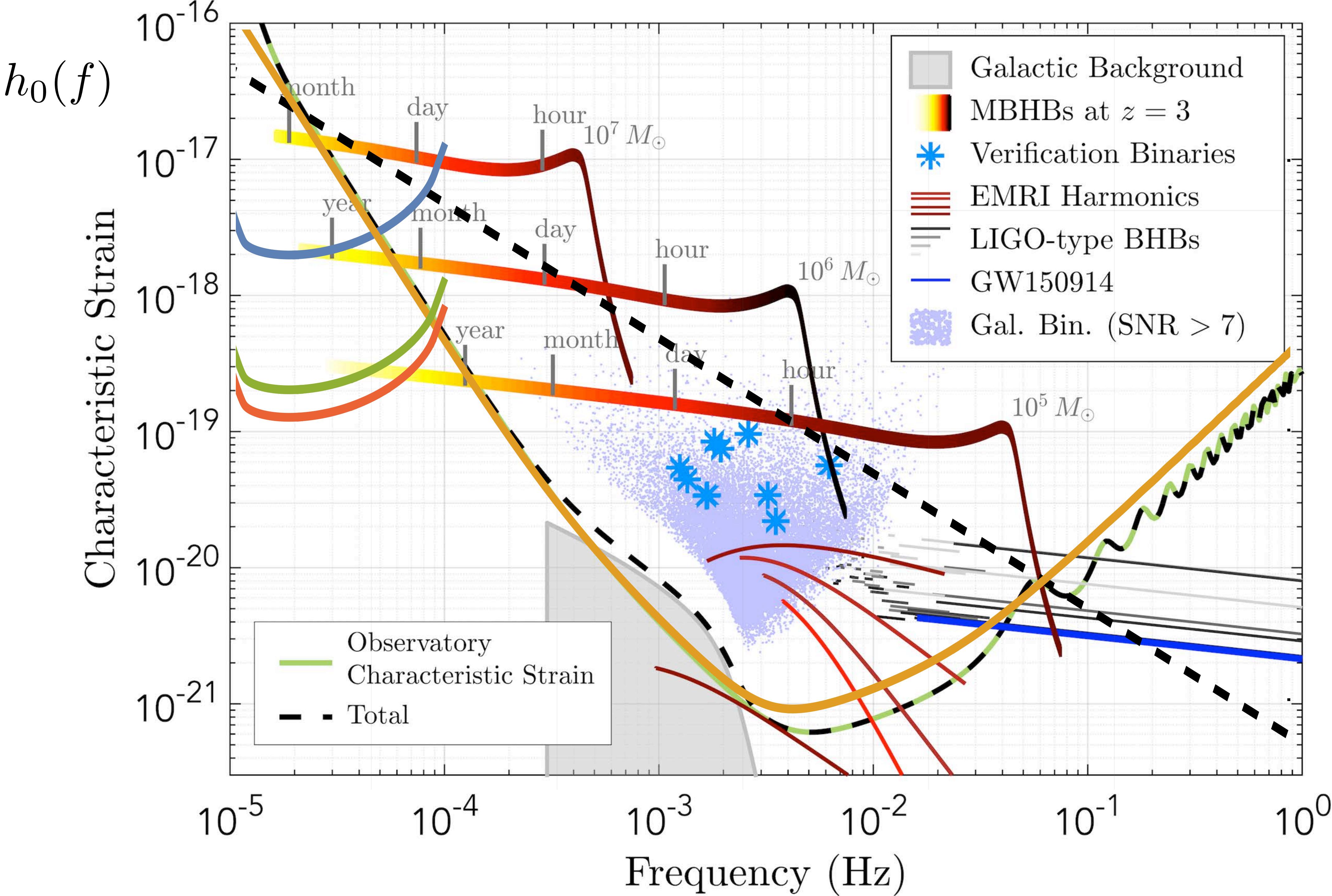
Semi-Major Axis (km)

9750.73

Eccentricity

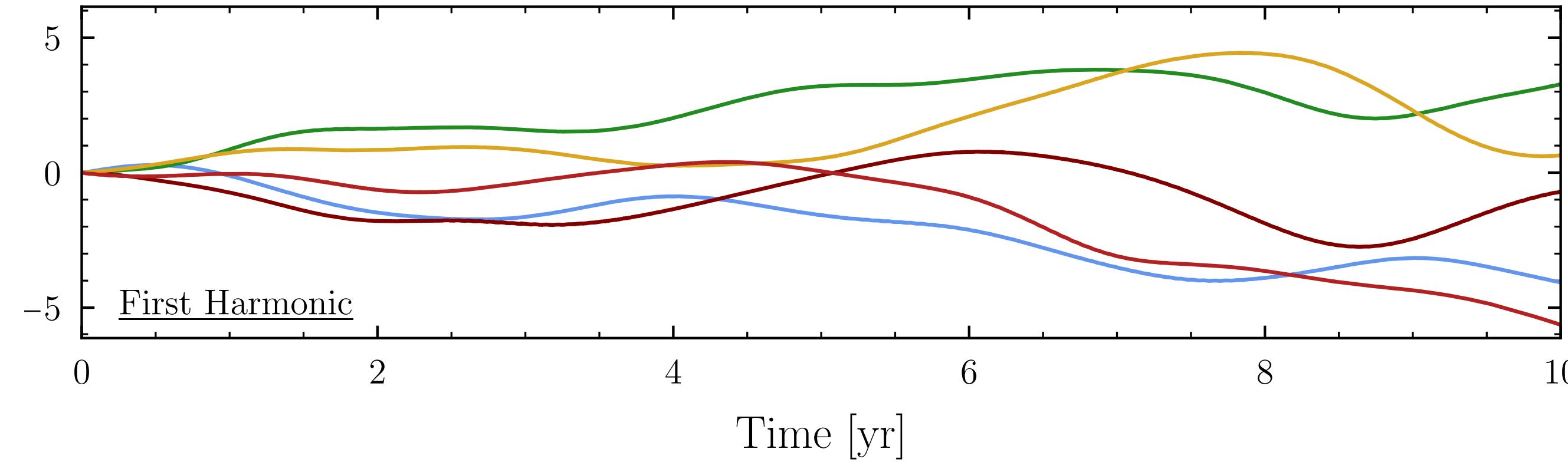
0.6383





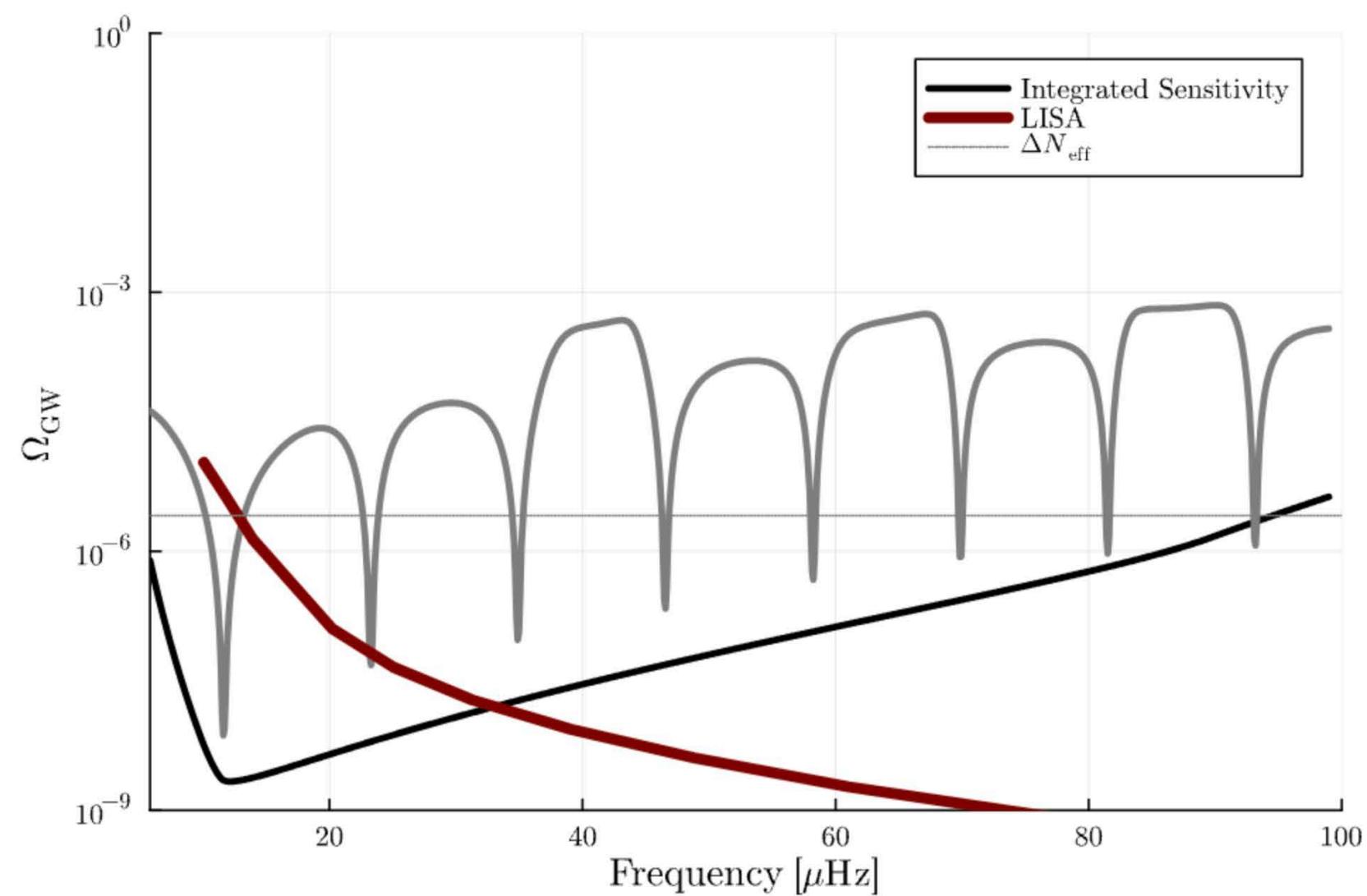


# Are we seeing GWs??



- Benefit of several orbits
- More data from several orbits

## Out of resonance?



# Detecting gravitational waves with SLR

- The  $\mu\text{Hz}$  band is very rich for **astrophysical** and **cosmological** sources
- The resonant **absorption of GWs by satellites** may give a handle



We are looking forward  
to collaboration  
with SLR/GNSS  
colleagues!

